THEORY AND PRACTICE

IN THE

DESIGN AND CONSTRUCTION OF DOCK WALLS.

BY

J. ROMILLY ALLEN,

ASSOC. INST. C.E.

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INTRODUCTION.

THE theoretical portion of this work is arranged in the form of a series of problems, bearing on the subject of dock walls, which, it is to be hoped, will put the engineering student in possession of most of the facts already ascertained and the deductions which have been made therefrom by mathematicians up to the present time. The information from which these problems have been compiled has been chiefly derived from the authorities hereafter mentioned.

Some of the geometrical investigations are, however, now published by the author for the first time.

The graphic method, heretofore chiefly applied to calculating the strains on roofs and girders, has been freely made use of to render the meaning of the algebraical formulæ more thoroughly intelligible.

In classifying different structures, dock walls may be said to occupy a situation intermediate between masonry dams and retaining walls, the former being built to support a pressure of water, and the latter to resist a thrust of earth. A dock wall has to perform both these functions.

In order to estimate the effect produced by the different forces which come into play in any structure, it is necessary to understand the nature of these forces, that is to say, to know their

$$\left.\begin{array}{l}\textbf{Magnitudes}\\ \textbf{Directions}\\ \textbf{Points of Application}\end{array}\right\}$$

The forces which come into play in the case of dock walls are three, viz.:

$$\left.\begin{array}{l}\textbf{1. Water Pressure}\\ \textbf{2. Earth Thrust}\\ \textbf{3. Weight of Masonry}\end{array}\right\}$$

Each of these will be considered separately, and finally their joint effect explained.

In the following pages the portion of wall under consideration is supposed to be a section one foot in length.

Water Pressure.

The laws which govern fluid pressure were first discovered by Archimedes, who lived B.C. 287 to 212.

The definition of a fluid is as follows:

A fluid is a substance, such that the mass of it may be very easily divided in any direction; and of which portions, however small, can be very easily separated from the whole mass.

The laws of fluid pressure are these, viz.:

The pressure of a fluid on any surface with which it is in contact is perpendicular to that surface.

The pressure at any point of a fluid is the same in every direction.

If a mass of fluid be at rest, any portion may be supposed to become rigid without affecting its equilibrium or the pressure of the surrounding fluid.

For any portion of a fluid mass may be contemplated as a separate body, surrounded by fluid, which presses perpendicularly at all points, and its solidification will introduce no change in the pressures upon it, and therefore no change in the pressure at any other point of the fluid.*

This last proposition enables the laws of statics to be applied to cases of the equilibrium of fluids.

Thus the pressure of water against the sloping face of a dock wall is determined by imagining the prism of fluid lying vertically over the face to become solid, and then calculating the effect produced by its weight on the principle of the inclined plane without friction.

* See Besant's 'Hydrostatics,' p. 9.

The pressure of water is thus reduced to question of statics. The thrust of earth is however of a more complicated kind, and it is only quite recently that its true nature has been fully understood,

Earth Thrust.

The tenacity of earth is so uncertain an element that it is neglected, and the earth merely considered as a granular mass having friction between its particles.

By a granular mass is meant a collection of very small solids more or less of the same size and shape, such, for instance, as a heap of sand or pile of shot.

If dry sand be poured out of a vessel with a spout on to a flat surface it will form a conical heap, the sides of which will make particular angle with the horizontal, and it will be found that the steepness of this slope can never be increased beyond a certain limit, however judiciously the sand be poured, or however carefully it is heaped up afterwards. This slope is called the angle of repose of the material of which the granular mass is composed, and varies for different substances, scarcely in any case exceeding 45°, and sometimes being as low as 20°.

It is plain, therefore, that a granular mass is not capable of maintaining a vertical face, except where there is cohesion as well as friction between the grains; and, as has been said above, the former is neglected in the following investigations.

Now, since a bank of earth is not able to remain vertical of its own accord, it is evident that when an upright wall is built against it a certain amount of sustaining power must be exerted by the wall to prevent some portion of the earth from slipping down.

If the wall was removed altogether the face of earth would assume its angle of repose, and it was accordingly at first thought that the earth thrust was produced by the whole wedge of earth between the back of the wall and the natural slope.

This, however, as will be seen hereafter, is not the case, since

when a wall gives way the earth does not break away along the line of natural slope, but along a plane of rupture half-way between the back of the wall and the angle of repose, so that the earth thrust is due to only about half the wedge of earth between the back of the wall and the natural slope.

M. de Coulomb and M. de Prony, who wrote at the beginning of this century, were the first to prove this mathematically, and since that time the subject has been more fully gone into by Neville and Rankine. A very good paper referring to this matter was read by Mr. Constable before the American Society of Civil Engineers, Jan. 15, 1873.

The question of walls constructed to resist a thrust of earth has been chiefly dealt with by military engineers, who are concerned with it in constructing fortifications.

The subject of earth thrust presents a good field for careful experiments on a large scale, which have yet to be tried.

Experiments with shot, peas, corn, &c., made with a box having glass sides and moveable end, would be valuable.

Weight of Wall.

There is but little to be said with regard to the third force in question, viz. the weight of masonry, except that it acts vertically downwards through the centre of gravity of the whole mass.

Joint Effect of Forces.

The combined effect of the three forces that have been considered is to produce certain strains within the structure.

Against the face of the wall is a pressure of water tending to push it backwards.

Against the back of the wall is a thrust of earth tending to push it forwards.

Thirdly, there is the weight of the masonry tending to counteract the two former.

If the water pressure and earth thrust are equal, or nearly so, they will neutralize each other when the dock is full. It is, however, necessary to construct a wall sufficiently strong to resist the earth thrust when the dock is empty.

The effect of the earth thrust and weight of masonry is to produce a resultant pressure cutting the base in a particular point, called the centre of pressure.

If the centre of pressure falls beyond the limits of the base the wall will be overturned; but any deviation of the centre of pressure from the centre of base produces additional strain on that side, and before the wall can be overthrown this strain will become so great as to crush the materials of which the structure is composed.

This may be easily verified by placing a heavy block of stone on a yielding surface and attempting to overturn it. Before the block can be overthrown a dent is produced in the yielding surface at the corner round which it turns. If the surface be unyielding the corner of the stone itself will be damaged. Thus the block of stone does not overturn round the extreme point of its base, but round some point within it, which corresponds to the centre of pressure alluded to above. It is necessary, therefore, that the distance between the centre of pressure and the centre of the base should never be so great as to allow of the possibility of the materials of the structure being crushed.

The general nature of the forces coming into play against dock walls and their joint effect having been now considered, the student is in a position to proceed to the problems, which reduce the results of theory to practical formulæ.

PREFACE.

THE theoretical portion only of this work is now ready for publication, but the author intends as soon as possible to supplement it by a practical treatise on the materials and appliances used, and the difficulties usually met with by engineers in the construction of dock works. There are but few books in our libraries devoted exclusively to the subject dealt with in the present volume; it is therefore hoped that the information collected together by the author may prove useful to the engineering student, who in the present day is required to be something more than the mere rule of thumb practical man, now happily disappearing before the rapid strides of scientific advancement. It must not, however, be forgotten that it is to practice that theory is indebted for all the data upon which her subsequent reasoning is founded, and that the accuracy of the results obtained will depend entirely on the soundness of these data. Experiments are also of great value in confirming the results of theory and practice combined, and it is much to be regretted that so few have been tried bearing on this subject.

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THEORY AND PRACTICE

IN THE

DESIGN AND CONSTRUCTION OF DOCK WALLS.

PART I.—THEORY.

THE present treatise is divided into two parts: the first embracing the theoretical considerations which should guide the engineer in the design of his work; and the second containing a full description of the way in which these principles are applied in practice.

The object held in view has been to collect together all the most important problems in connection with the Theory of the Design of Dock Walls, and to present them in as simple a form as possible, omitting at the same time no step in the reasoning which may be necessary to render the process complete in itself.

The mechanical problems are all worked out by means of the triangle of forces; and in calculating the maximum thrust of earth the differential calculus is avoided by the introduction of a new geometrical demonstration, requiring a knowledge of 'Euclid's Elements' only.

The following are the various authorities which have been consulted:

Prony's 'Poussée des Terres.' Paris. 1802.
Rankine's 'Civil Engineering,' p. 401 to p. 413.
Spons' 'Dictionary of Engineering,' p. 2728 to p. 2749.
Moseley's 'Engineering,' p. 443 to p. 461.
Twisden's 'Mechanics,' p. 162 to p. 173.
Coulomb's 'Théorie des Machines Simples.' Paris. 1779.
Dobson's 'Art of Building,' p. 12 to p. 22.
Neville on 'Pressure of Earth.'
'Building News,' volume for year 1873, p. 421, &c.
'Encyclopædia Metropolitana,' vol. iii., p. 71 to p. 83.
'Penny Cyclopædia,' article "Revetment."

LIST OF PROBLEMS.

Problem I.—To find the amount, direction, and point of application of a fluid pressure acting against the plane face of a wall.

Problem II.—To find the amount, direction, and point of application of the thrust of earth against the vertical back of a wall.

Problem III.—To find the wedge of maximum earth thrust.

Problem III.—Notes.

Table showing how earth thrust varies according to the value of angle of repose.

Investigation of the general case of earth thrust, when back of wall is not vertical.

Earth thrust produced by surcharge.

Problem IV.—To find the resultant pressure produced by the earth thrust and weight of wall, and to calculate its effect.

Problem V.—To transform a rectangular profile into one of equal strength, having a straight sloping face.

Problem VI.—To find the position of the centre of gravity of the section of a wall having a vertical back and a straight sloping face.

Problem VII.—To find the alteration in position of centre of gravity of a wall, produced by stepping at the back.

Problem VIII.—To find the alteration in position of centre of gravity of a wall, produced by giving its face a curved batter in place of a straight one.

Problem IX.—To compare the stability of a wall of rectangular section with that of one of the same sectional area, having a straight sloping face.

Problem X.—To show how uniformly varying pressure may be represented graphically by either a triangle or a trapezium; the *centre of pressure* lying vertically under the *centre of gravity* of the figure.

Plate 1.

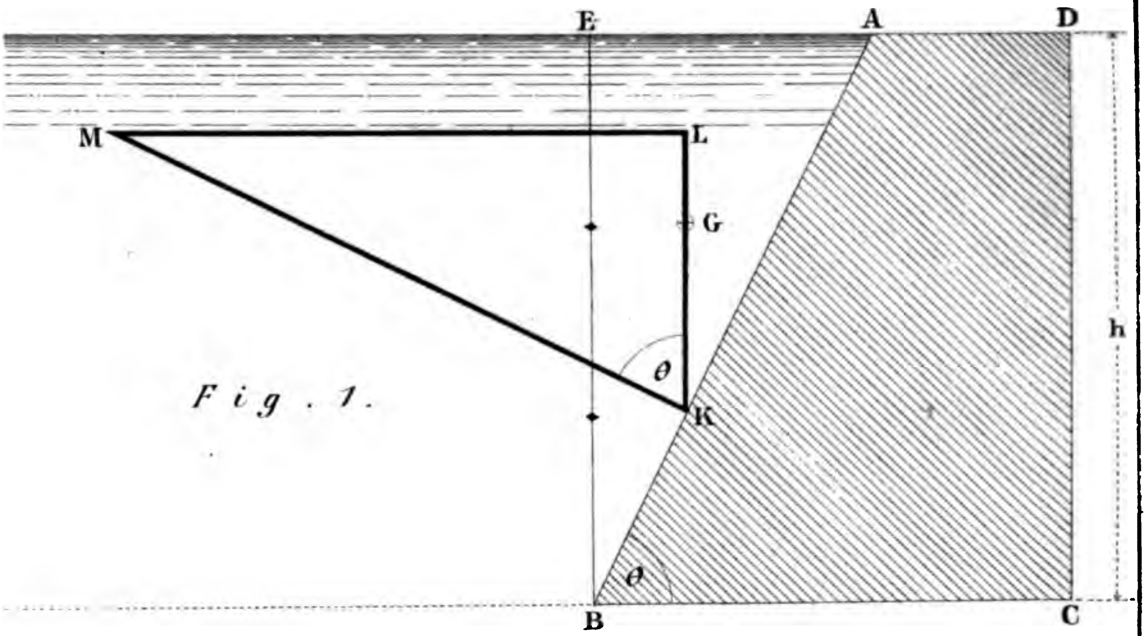
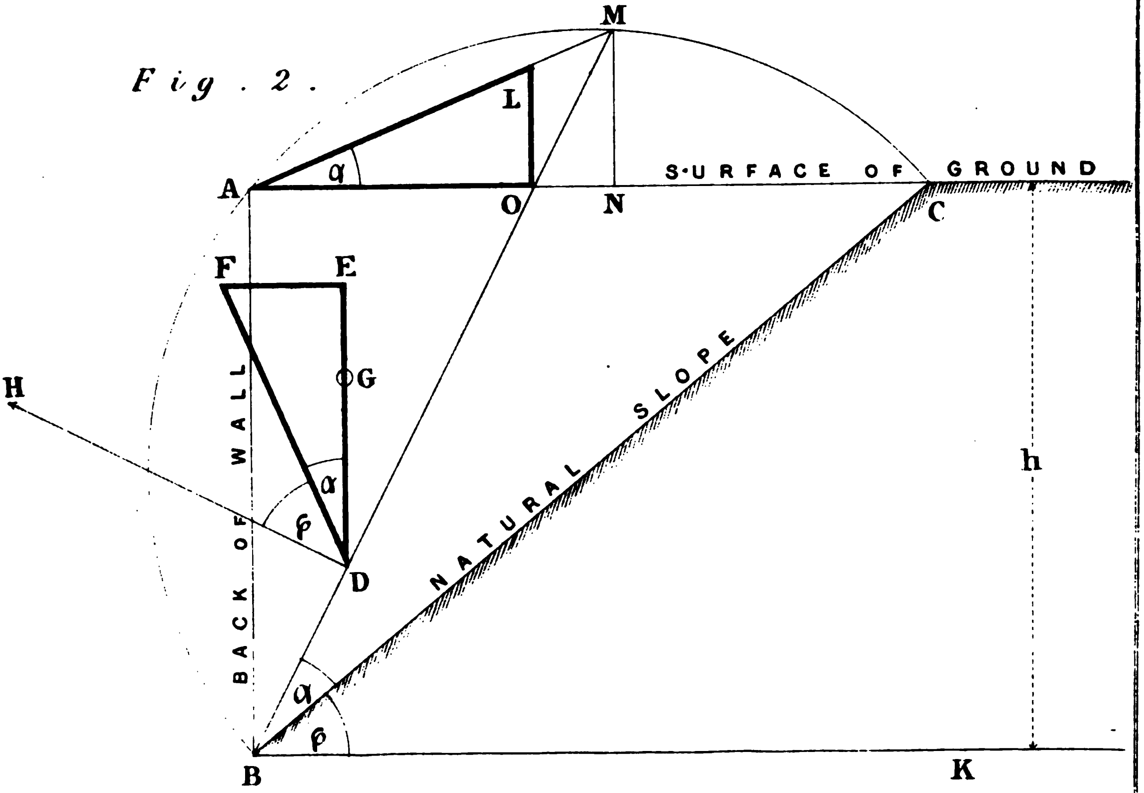


Fig. 1.

Fig. 2.



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Problem XI.—Given an uniformly varying pressure acting on a rectangular surface and the amount of the deviation of the *centre of pressure* from the *centre of gravity* of that surface—

Find the maximum pressure produced in terms of the deviation and of the mean pressure.

Problem I.

To find the amount, direction, and point of application of a fluid pressure acting against the plane face of a wall.

A B C D (Fig. 1) represents the profile of a wall 1 foot in length, having a straight battering face A B, which supports the pressure of a fluid whose surface is level with the top of wall A D.

Through B draw a vertical plane E B, and suppose the fluid contained in A B E to become solid, being supported, without friction, by the two planes A B, B E.

In order to find the required pressure which will act normally to A B, draw the triangle of forces thus:

1. Through G (the centre of gravity of prism A B E) draw L K vertical and equal to A E, to represent the *weight of* A B E.
2. Through L draw L M horizontal, to represent the *reaction of plane* E B.
3. Through K draw K M at right angles to face of wall A B, to represent the *required pressure* acting normally to its surface.

Now, let

Depth of fluid supported by wall	=	h
Angle which face of wall makes with horizon	=	θ
Weight of 1 cubic foot of fluid	=	w
Total pressure along 1 foot in length of wall face	=	P

By producing M K to cut B C it may easily be shown that

$$\text{Angle } MKL = ABC = \theta.$$

So that

$$\text{Required pressure } MK = LK \sec.\theta$$
$$= \text{weight of } ABE \times \sec.\theta.$$

Therefore

$$= \tfrac{1}{2} w h^2 \cot.\theta \times \sec.\theta.$$

$$\textbf{1.}\quad \mathbf{P = \tfrac{1}{2}\, w\, h^2 \,\text{cosec.}\,\theta.}$$

When A B is vertical, $\theta = 90°$, and cosec. $\theta = 1$.

$$\textbf{2.}\quad \mathbf{P = \tfrac{1}{2}\, w\, h^2.}$$

Also, since L K passes through G, the centre of gravity of A B E,

$$BK = \tfrac{1}{3} AB,$$

3. $\left\{\begin{array}{l}\textbf{Depth of centre of pressure K below sur-}\\ \textbf{face of fluid} \;\; \ldots\ldots\ldots\ldots\ldots\ldots\end{array}\right\} = \tfrac{2}{3}\mathbf{h}$

Fluid pressure acts normally to face of wall.

Problem II.

To find the amount, direction, and point of application of the thrust of earth against the vertical back of a wall.

The surface of the ground is supposed to be horizontal and level with the top of the wall. Friction between the earth and the back of wall is neglected.

A B (Fig. 2) represents the vertical back of a wall of rectangular section 1 foot in length.

B C is the natural slope of the earth supported, making an angle C B K $= \phi$ with the horizontal.

It is now assumed that whatever thrust there is behind the wall is produced by a wedge of earth, such as A B C or A B O, acting in one solid mass, having a tendency to slide down the plane B C or B O on which it rests.

There are two cases to be considered, namely:

1.—Where there is supposed to be *no friction* between the wedge and the plane which supports it.

Here the tendency of the prism A B C to produce horizontal thrust against the back of the wall *increases* with the inclination of B C to the horizon, whilst the weight of A B C *decreases* exactly in the same proportion.

The horizontal pressure against A B is therefore a constant quantity, and is equal in amount, direction, and point of application to that of a fluid of the same specific gravity as earth.

This may easily be verified by referring to Problem I., where the investigation is quite similar.

So if friction be neglected, and

Horizontal earth thrust against vertical back of wall per foot forward	=	**T**
Height of back wall	=	**h**
Weight of 1 cubic foot of earth	=	$\mathbf{w_1}$

4. $\mathbf{T = \frac{1}{2} w_1 h^2}$

5. Depth of centre of pressure below surface $= \frac{2}{3}\mathbf{h}$

Case 2.—Where *friction is* supposed to exist between the wedge and the plane which supports it.

The effect of friction is to reduce the horizontal thrust by counteracting the tendency of the wedge to slide.

In the case of the wedge A B C (acting as one solid mass) where B C is the natural slope, the amount of friction will be just sufficient to prevent sliding altogether, and there will be *no horizontal thrust* at all.

The same reason of course applies to any prism resting on a plane making an angle less than ϕ with the horizon.

Any horizontal pressure, therefore, which exists behind the wall must be due to a wedge of earth, the lower surface of which makes an angle *greater than* ϕ with the horizon.

Let B O (Fig. 2) be any plane making an angle O B K with B K greater than the natural slope of earth.

In order to find the amount of horizontal thrust caused by prism

A B O against the back of the wall, construct the triangle of forces, thus:

1. Through G (the centre of gravity of A B O) draw D E vertical and equal to A O, to represent the *weight of wedge* A B O.

2. Through D draw D F, making angle H D F = ϕ with the normal D H to B O, to represent the *reaction of plane* B O.

3. Through E draw E F horizontal, to represent the *required thrust* behind wall.

Let

Horizontal thrust behind vertical back of wall, taking friction into account	=	**H**
Height of back of wall A B	=	**h**
Angle of repose of earth C B K	=	ϕ
Angle C B O which B O makes with natural slope	=	α
Weight of 1 cube foot of earth	=	$\mathbf{w_1}$

By producing H D, F D, and E D to cut B K, it may easily be shown that

$$\text{Angle FDE} = \text{OBC} = \alpha.$$

And E D was made by construction = A O, to represent weight of prism A B O, which varies as A O (the height A B being constant).

So that

$$\text{Required horizontal thrust EF} = \text{ED tan. } \alpha.$$

$$\textbf{6.}\quad \mathbf{H} = \textbf{weight of prism ABO} \times \textbf{tan. } \alpha.$$

$$= \tfrac{1}{2}\, w_1\, h\, .\, \text{AO} \times \frac{\text{EF}}{\text{ED}}$$

$$= \tfrac{1}{2}\, w_1\, h\, .\, \text{AO} \times \frac{\text{EF}}{\text{AO}}$$

$$\textbf{7.}\quad \mathbf{H} = \tfrac{1}{2}\, \mathbf{w_1\, h\, .\, EF}.$$

Here, however, E F is not a constant quantity, but varies with the inclination of B O or with angle α.

For instance, when B O and B C coincide, angle α = O, and consequently E F, and therefore H = O, as stated previously.

Now if the slope of B O be gradually increased beyond ϕ,

The tendency of the prism A B O to cause horizontal thrust against back of wall by slipping down plane B O *becomes greater*.

But at the same time, the weight of A B O (which varies as A O) *becomes less*.

The amount of friction (which depends on the weight of A B O) *becomes less* also.

These varying quantities combine to form a *maximum value* for H or E F, which represents the earth thrust at some particular inclination of B O.

This inclination is determined in the following Problem.

Problem III.

To find at what slope of B O the earth thrust represented by E F attains its maximum value.

On B C (Fig. 2) describe a semicircle B A C.

Produce B O to cut it in M.

Join A M.

Draw O L, M N at right angles to A C.

Now, because angle M A C = M B C in same segment

$$= a$$

$$= \mathrm{FDE}$$

and D E was made equal to A O.

Therefore triangle A O L is in every respect similar and equal to the triangle of forces D E F.

So that E F representing thrust against back of wall is equal to L O.

Therefore the thrust will be greatest when L O is a maximum.

Now, since A B, L O, M N are parallel to each other, therefore from similar triangles,

$$\frac{\mathrm{LM}}{\mathrm{LO}} = \frac{\mathrm{AM}}{\mathrm{AB}}$$

or

$$\mathrm{ML} = \frac{\mathrm{MA}}{\mathrm{AB}} \times \mathrm{LO}$$

Also

$$\frac{LO}{LA} = \frac{MN}{MA}$$

$$LO = \frac{MN}{MA} \times LA$$

$$= \frac{MN}{MA} \times (MA - ML)$$

$$= MN - \frac{MN}{MA} \times ML$$

Substituting value of M L,

$$= MN - \frac{MN}{MA} \times \frac{MA}{AB} \times LO$$

$$= MN - \frac{MN}{AB} \times LO$$

$$LO\left(1 + \frac{MN}{AB}\right) = MN$$

$$LO = \frac{AB \times MN}{AB + MN}$$

Now A B is a constant, so that if successive values be given to M N in the above fraction, the numerator will increase much more rapidly than the denominator.

Therefore L O is greatest when M N is greatest.

But M N is greatest when M bisects arc A C.

Therefore E F, which represents the earth thrust and is equal to L O, attains its maximum value when slope B O bisects the complement of ϕ (the angle of repose).

Or when

$$\text{Angle } \alpha = \tfrac{1}{2}(90^\circ - \phi)$$

$$= \left(45^\circ - \frac{\phi}{2}\right)$$

$$= \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$

Referring back to Problem II., Formula 6, it will be found that

Horizontal thrust produced by prism A B O = weight of A B O $\times$ tan. α.

But when this thrust attains its maximum value,

$$\text{Angle ABO} = \text{OBC} = a = \tfrac{1}{2}(90^\circ - \phi);$$

therefore

$$\text{Weight of prism ABO} = \tfrac{1}{2} w_1 h^2 \tan. a,$$

and

$$8. \quad \textbf{Maximum earth thrust} = \tfrac{1}{2} w_1 h^2 \tan.^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right).$$

This is the same formula as that obtained by Moseley and Twisden.

Rankine puts the formula in this form:

$$9. \quad \textbf{Maximum earth thrust} = \tfrac{1}{2} w_1 h^2 \frac{1 - \sin. \phi}{1 + \sin. \phi},$$

where $\frac{1 - \sin.\phi}{1 + \sin.\phi}$ is merely a trigonometrical reduction of $\tan.^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$.

Since E D passes through centre of gravity of prism A B O, therefore

10. Depth of centre of pressure below surface = $\tfrac{2}{3}$ h.

The pressure acts normally to back of wall.

Notes on Problem III.

It has just been shown that the maximum earth thrust against the back of a vertical wall = weight of 1 cub. feet of earth × $\tfrac{1}{2}$ the square of the height of wall × $\tan.^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$.

The following table gives the values of $\frac{\pi}{4} - \frac{\phi}{2}$, and also of $\tan.^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$ for different inclinations of ϕ.

TABLE I.

ϕ	$\frac{\pi}{4} - \frac{\phi}{2}$.	$\tan.^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$.	ϕ	$\frac{\pi}{4} - \frac{\phi}{2}$.	$\tan.^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$.
°	° ′		°	° ′	
0	45 0	1·000	40	25 0	·217
5	42 30	·839	45	22 30	·172
10	40 0	·704	50	20 0	·132
15	37 30	·588	60	15 0	·071
20	35 0	·490	70	10 0	·031
25	32 30	·405	80	5 0	·007
30	30 0	·333	90	0 0	·000
35	27 30	·271			

This table is represented graphically on the accompanying diagram (Fig. 3), by means of which the earth thrust against the back of a vertical wall, for any particular value of ϕ may be found thus:

Draw the line A C making an angle B A C = ϕ with the horizontal line A B and cutting the curve in the point C.

Then

$$\text{Thrust along 1 foot in length of wall} = \text{A C} \times \tfrac{1}{2} w_1 h^2.$$

The line A B on diagram is 10 inches long, and represents the greatest thrust that can come against the back of wall, namely, when the friction between the particles of earth is entirely destroyed and $\phi = 0$. Calling A B unity or 1·000 the length of any other line such as A C, cutting curve may be measured to three places of decimals with a scale 100 = 1 inch.

The diagram illustrates very forcibly the rapidity with which the pressure *increases* as the value of ϕ the angle of repose *decreases*, especially when it is less than 45°.

Thrust when ϕ = 0	is about	twice	thrust	when ϕ	= 20°
"	"	"	5 times	"	"	= 42°
"	"	"	10 times	"	"	= 55°
"	"	"	140 times	"	"	= 80°

Showing what economy of masonry section may be effected by careful filling behind the wall, and also how easily failure may occur when this is neglected.

The only case of earth thrust, which has been considered in the preceding pages, is that of earth having a plane *horizontal* surface acting against the *vertical* back of a wall.

The general case is determined in the following manner:

B C (Fig. 4) is the natural slope of earth supported by back of wall A B (not vertical).

A C is the surface of ground (not horizontal).

It has been shown in Problems II. and III. that the earth thrust is *not* produced by the wedge A B C, but by some other prism, such as

Plate 2.

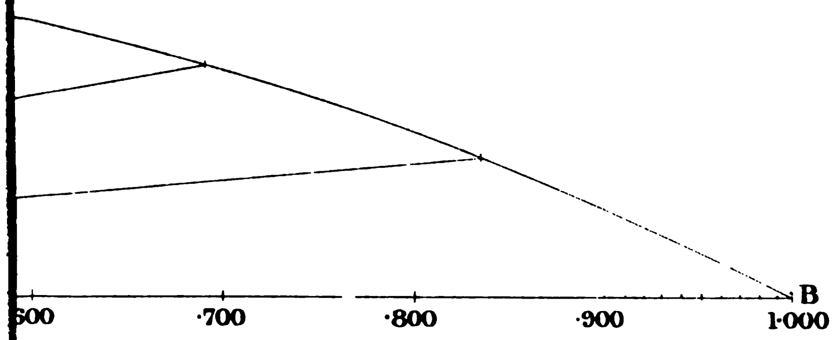


Fig. 4.

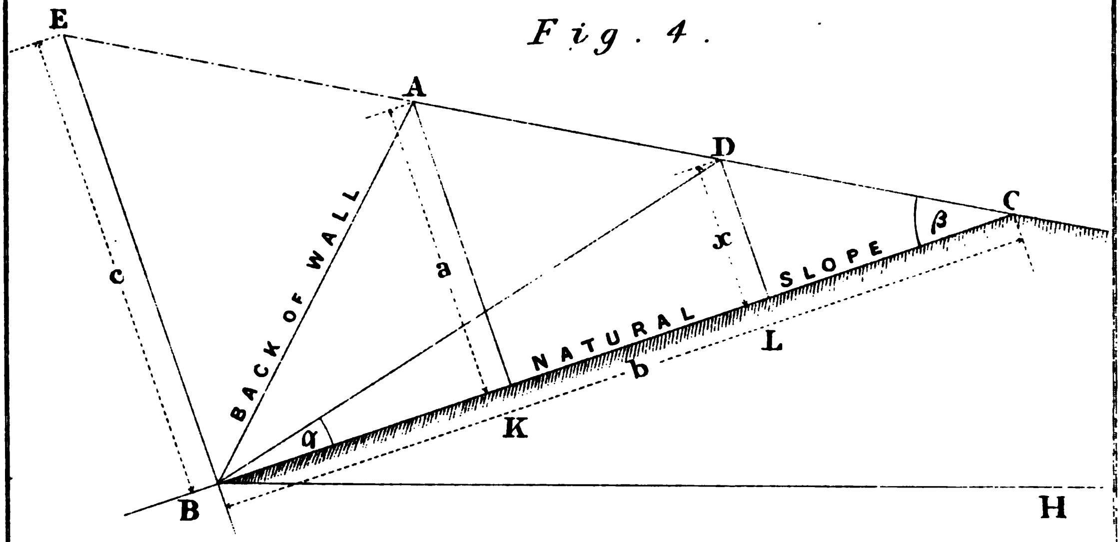
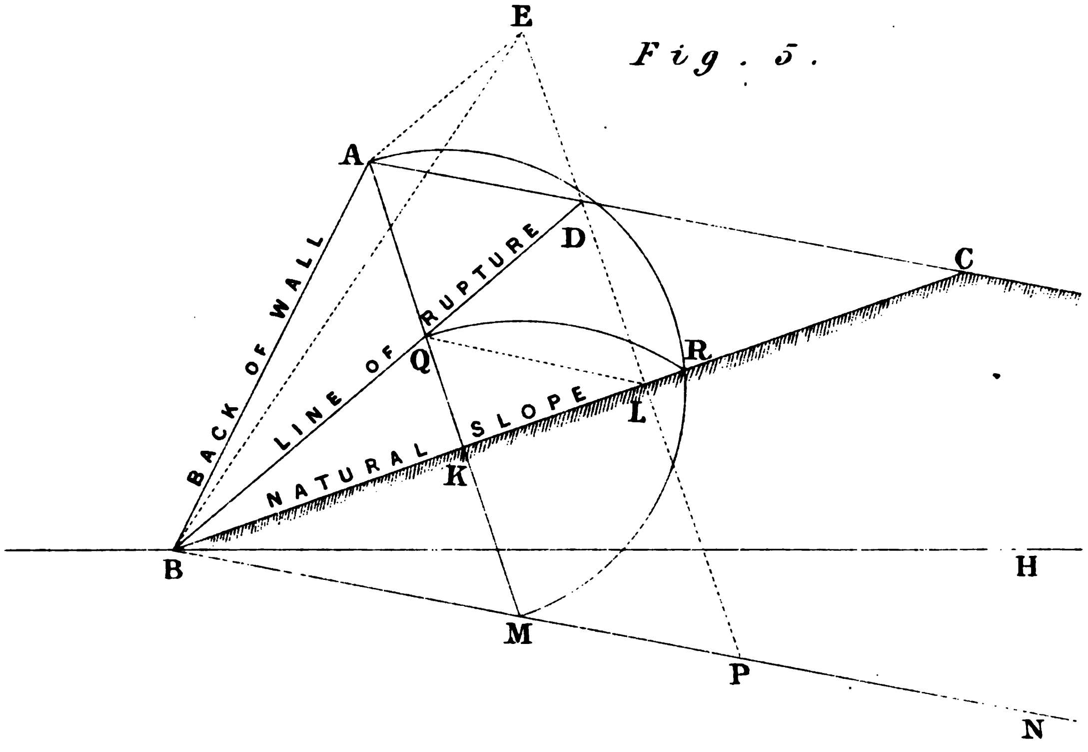


Fig. 5.



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A B D resting on a plane B D, inclined to the horizon at an anglé D B H, greater than ϕ, the angle of repose of earth.

A triangle of forces, exactly similar to that shown on Fig. 2, may be drawn in this instance also, since the directions of the lines representing the different pressures are independent either of the slope of the back of the wall or of the slope of the ground behind it.

The weight of the prism A B D will of course be different in this case, which fact is taken into account in the demonstration which follows.

Accordingly, as in Problem II., Formula 6,

$$\text{Earth thrust due to prism} = \text{weight of prism ABD} \times \tan. \alpha.$$

From points A and D let fall perpendiculars A K, D L on B C.

$$\left.\begin{array}{rl} \textbf{Let} \quad \mathbf{AK} & = \mathbf{a} \\ \mathbf{BC} & = \mathbf{b} \\ \textbf{Angle ACB} & = \beta \end{array}\right\} \textbf{ Constant quantities.}$$

$$\left.\begin{array}{rl} \mathbf{DL} & = \mathbf{x} \\ \textbf{Angle DBC} & = \alpha \end{array}\right\} \textbf{ Variable.}$$

It is required to ascertain when area A B D $\times$ tan. α attains its maximum value.

$$\begin{aligned} \text{Area ABD tan. } \alpha &= (\tfrac{1}{2} a b - \tfrac{1}{2} x b) \times \tan. \alpha \\ &= \tfrac{1}{2} b\,(a - x) \times \frac{x}{b - x \cot. \beta} \\ &= \tfrac{1}{2} b \times \frac{a x - x^2}{b - x \cot. \beta} \end{aligned}$$

Differentiating the quantity $\dfrac{a x - x^2}{b - x \cot. \beta}$, and putting its value $= 0$ for a maximum, the following equation for x is obtained:

$$(b - x \cot. \beta)(a - 2x) - (a x - x^2)(- \cot. \beta) = 0$$

$$\textbf{11.} \quad \mathbf{x}^2 \cot. \beta - 2\,\mathbf{b}\,\mathbf{x} = -\,\mathbf{a}\,\mathbf{b}$$

$$\begin{aligned} a b - b x &= b x - x^2 \cot. \beta \\ &= x\,(b - x \cot. \beta) \\ &= x \times \text{BL} \end{aligned}$$

$$\therefore \text{ Area ABC} - \text{area DBC} = \text{area DBL}$$

$$\therefore \text{ Area ABD} = \text{area DBL}.$$

Therefore

Earth thrust behind wall is greatest when area of triangle A B D = area of triangle D B L.

But

Earth thrust = area A B D tan. $a \times w_1$,

so

Maximum earth thrust = area D B L tan. $a \times w_1$,

or

$$13. \quad \textbf{Maximum earth thrust} = \tfrac{1}{2}\, w_1\, x^2.$$

The value of x found by solving quadratic equation 11 is

$$14. \quad x = b \tan. \beta - \sqrt{b \tan. \beta\, (b \tan. \beta - a)}.$$

At point B (Fig. 4) draw B E at right angles to B C and produce C A to meet it in E.

Let

$$B E = c.$$

Now

$$b \tan. \beta = c,$$

therefore

$$15. \quad x = c - \sqrt{c\,(c - a)}$$

and

$$16. \quad \textbf{Maximum earth thrust} = \tfrac{1}{2}\, w_1 \left\{c - \sqrt{c\,(c - a)}\right\}^2.$$

A geometrical method of drawing B D so that it shall divide the figure A B L D into two equal triangles, the angle B L D being a right angle, is shown on Fig. 5, and is demonstrated thus,

Draw B N parallel to the surface of the earth A C.
Draw A K M cutting B C at right angles in K and meeting B N in M.
On A M describe semicircle cutting natural slope B C in R.
Make M Q = M R.
Now B D drawn through Q is the required line.
Draw D L at right angles to B C.
Draw A E parallel to B D meeting L D produced in E.

$$\therefore \text{ Triangle BED} = \text{triangle BAD}$$
$$= \text{triangle BDL}$$

$$\therefore \text{ DL} = \text{DE}$$
$$= \text{AQ}$$

$\therefore$ Q L is parallel to A C or B N.

$\therefore$ By similar triangles,

$$\frac{\text{MQ}}{\text{QA}} = \frac{\text{BQ}}{\text{QD}}$$
$$= \frac{\text{BK}}{\text{KL}}$$
$$= \frac{\text{MK}}{\text{KQ}}$$
$$\text{MQ}(\text{MQ} - \text{MK}) = \text{QA}\,.\,\text{MK}$$
$$\text{MQ}^2 = \text{MK}(\text{MQ} + \text{QA})$$
$$= \text{MK}\,.\,\text{MA}$$
$$\therefore \text{ MR}^2 = \text{MK}\,.\,\text{MA}$$

which verifies the construction of describing a semicircle round the points M R A, and making M Q = M R. See *Euclid*, Book VI., Prop. VIII.

It has been shown in Problem III. that, if A B C be the prism of earth between the back of a wall and the line of natural slope, then the pressure produced by any other prism such as A B O may be found thus:

Round triangle A B C describe a circle (Fig. 6).
Produce B O to meet circle in M.
Join A M.
Draw O L at right angles to A C cutting A M in L.

This makes triangle A O L similar to the triangle of forces, and

Thrust produced by prism A B O = $\frac{1}{2}\, w_1\, h \times$ O L.

By taking a series of points, such as M in arc A M C, and repeating the above construction, the locus of the point L may be traced.

The curve which is drawn on Fig. 6 shows how the thrust varies for different wedges of earth, any vertical ordinate such as O L indicating the magnitude of the pressure due to particular prism A B O.

17. Maximum earth thrust – highest ordinate to curve A L O = ½ w, h.

B O (Fig. 6) is shown in the position which gives the greatest value for O L, and consequently prism A B O is that which produces the maximum thrust behind wall.

The geometrical construction previously explained (see Fig. 5) is here repeated to show that it agrees with the graphic method. In order to find the highest ordinate O L it is not necessary to draw the whole curve A L C, a few points near the middle being quite sufficient.

The plane O B on which the *prism of maximum thrust* rests is sometimes called either the *plane of rupture* or *plane of separation*.

When back of wall is vertical and surface of earth horizontal, plane of rupture bisects angle A B C between back of wall and natural slope.

When back of wall slopes towards bank, plane of rupture falls on side of line bisecting angle A B C nearer bank. When back of wall slopes away from bank, plane of rupture falls on side of line bisecting angle A B C nearer wall.

Surcharge.

[illegible]

Fig. 6.

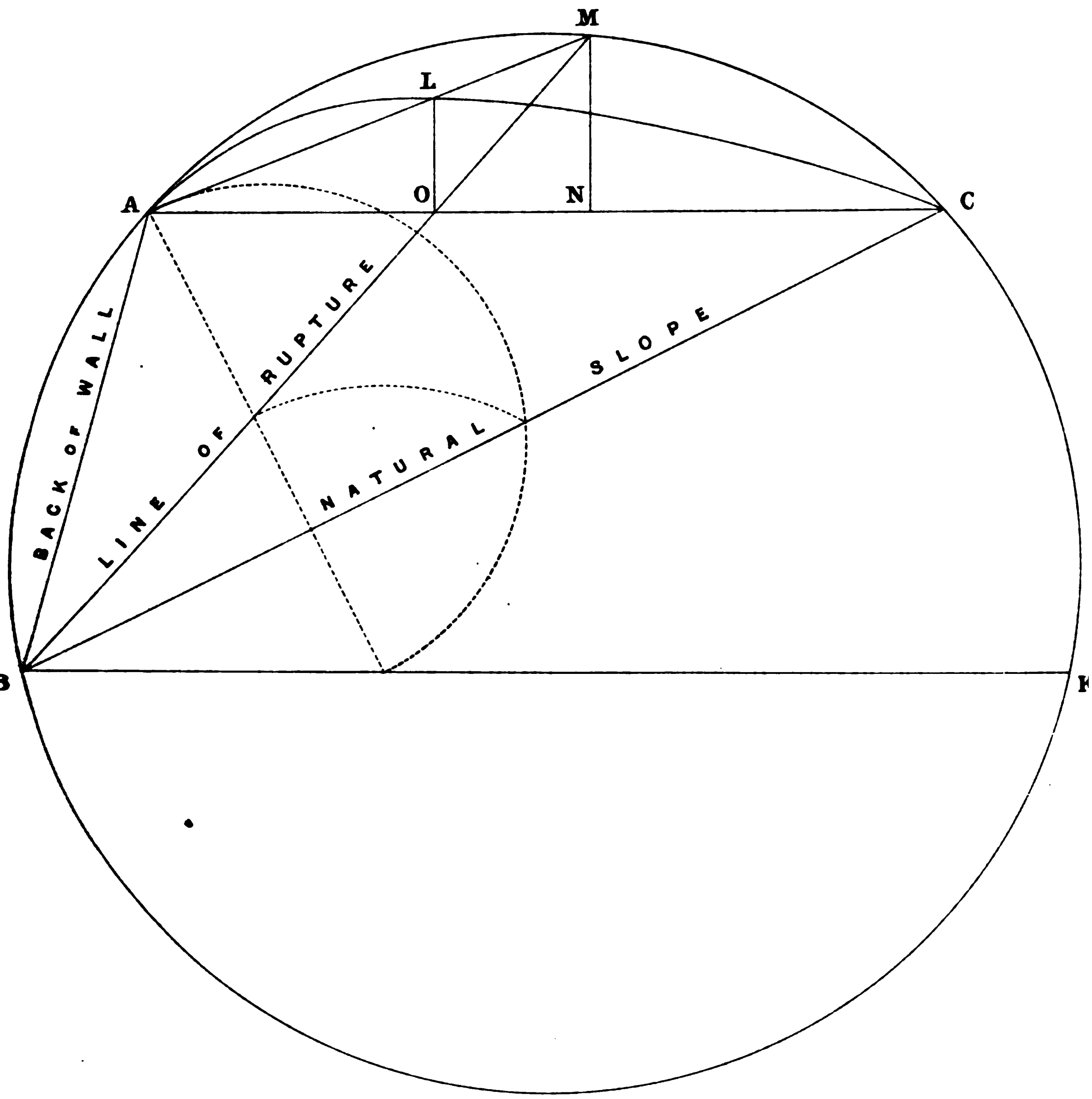

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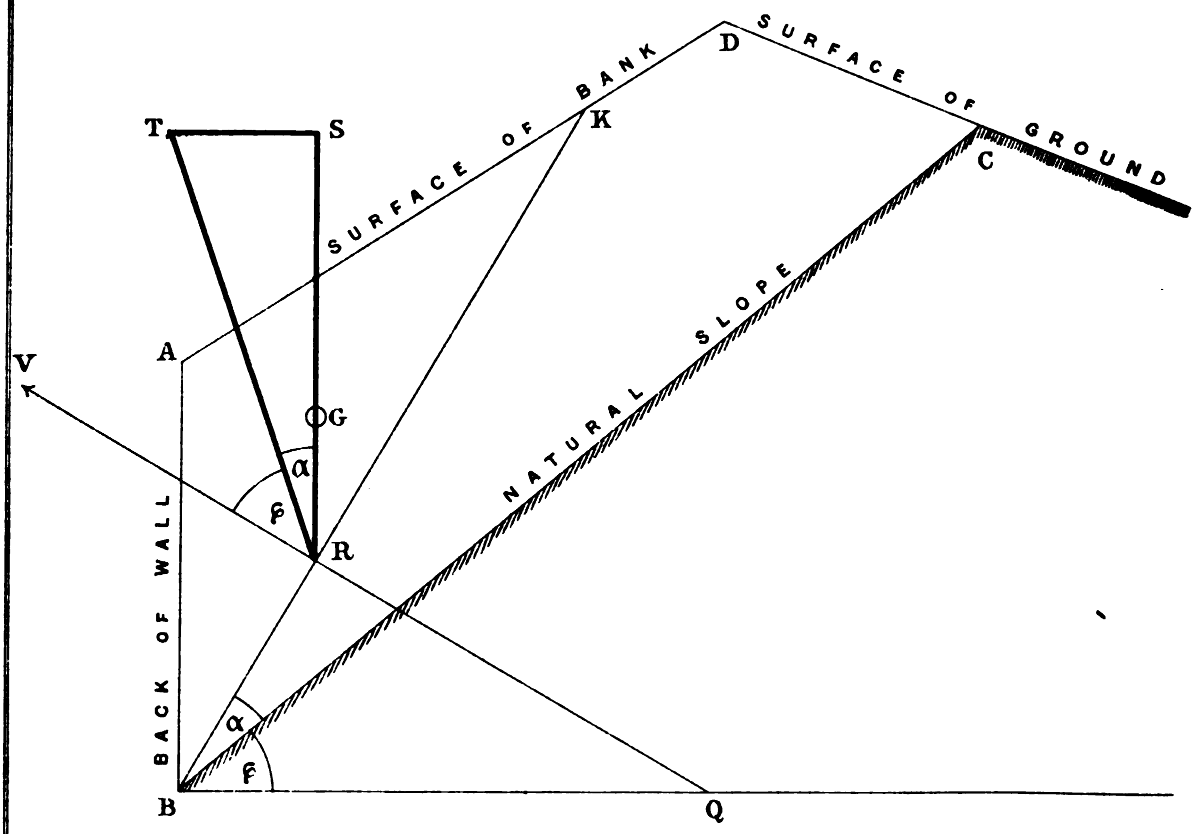
Fig. 7.



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but the formulæ obtained are somewhat unwieldy. The following method is graphic and may be understood without a knowledge of the differential calculus, which is indispensable if the question be treated algebraically.

A B (Fig. 7) is the back of a wall supporting a trapezoid of earth A B C D; B C being the natural slope.

The whole of the trapezoid A B C D does not act in producing horizontal thrust against A B, but only some portion of it, such as A B K resting on a plane B K making an angle K B Q greater than angle of repose of earth with the horizon.

This is evident from the following consideration, namely, that the earth tending to thrust outwards, being confined, acts more or less as one solid mass, and this being the case, the trapezoid A B C D can produce no thrust, since it is just prevented from sliding down B C by the friction of that surface. At any greater slope, such as B K, the friction is not sufficient to prevent the mass producing horizontal thrust against A B by sliding bodily downwards.

So let

$$\left.\begin{array}{ll}\textbf{Angle of repose of earth C B Q} & = \phi \\ \textbf{Angle K B C} \;\; .. \;\; .. \;\; .. \;\; .. \;\; .. & = \alpha\end{array}\right\}$$

To find the horizontal thrust of A B K draw the triangle of forces thus:

1. Through G, the centre of gravity of A B K, draw S G R vertical to represent the *weight of* A B K.
2. Draw R T making an angle V R T = ϕ with normal R V to B K to represent the *reaction of surface* B K.
3. Draw S T horizontal, which will give the *required thrust* against A B.

By producing V R to cut B Q it will easily be seen that angle V R S = R B Q;

$$\therefore \text{ Angle T R S} = \alpha.$$

So that

$$\begin{aligned}\text{Thrust S T} &= \text{R S tan. } \alpha \\ &= \text{weight of A B K tan. } \alpha.\end{aligned}$$

This fact enables the way in which the amount of the horizontal

earth thrust S T varies for different masses of earth, such as A B K, to be represented graphically thus:

> Join B D (Fig. 8).
> Draw A E at right angles to B D and produce it to P cutting B C in H.
> Draw K M and C P parallel to B D.
> Round triangle A B H describe a circle cutting B K in L.
> Join A L cutting K M in N.

Now

$$\begin{aligned}\text{Area ABK} &= \text{area ABD} - \text{area KBD} \\ &= (\text{AE} - \text{ME}) \times \tfrac{1}{2}\text{BD} \\ &= \tfrac{1}{2}\text{AM} \times \text{BD}.\end{aligned}$$

And since B D is constant A M may be taken to represent the varying weight of A B K.

Again,

$$\begin{aligned}\text{Angle LAH} &= \text{LBH in same segment} \\ &= \alpha;\end{aligned}$$

so that

$$\begin{aligned}\text{MN} &= \text{AM tan. } \alpha \\ \tfrac{1}{2} w_1 \times \text{MN} \times \text{BD} &= \tfrac{1}{2} w_1 \times \text{AM} \times \text{BD tan. } \alpha \\ &= \text{weight of ABK tan. } \alpha.\end{aligned}$$

If the above construction be repeated for a series of slopes B K the locus of the point N is obtained, being the curved line shown on diagram (Fig. 8).

The ordinate M N to curve drawn at right angles to A P represents the earth thrust for the slope B K.

Therefore

18. **Maximum earth thrust against A B acting horizontally at a point $\frac{1}{3}$ A B above B = longest ordinate to curve A N P drawn at right angles to A B, $\times$ B D $\times$ weight of 1 cubic foot of earth.**

In order to find the longest ordinate it will not be necessary to draw the whole curve, a few points between A and H being quite sufficient.

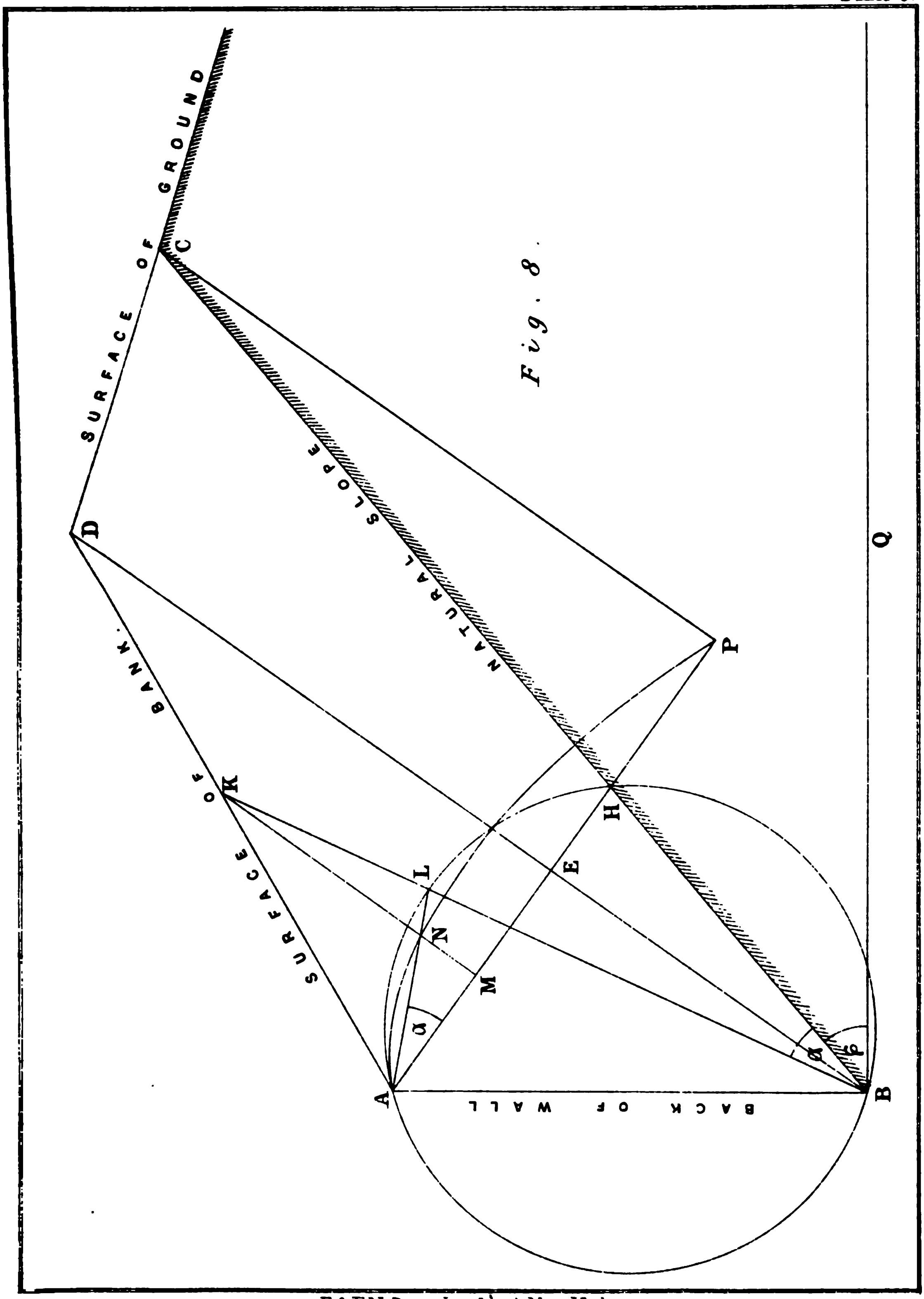

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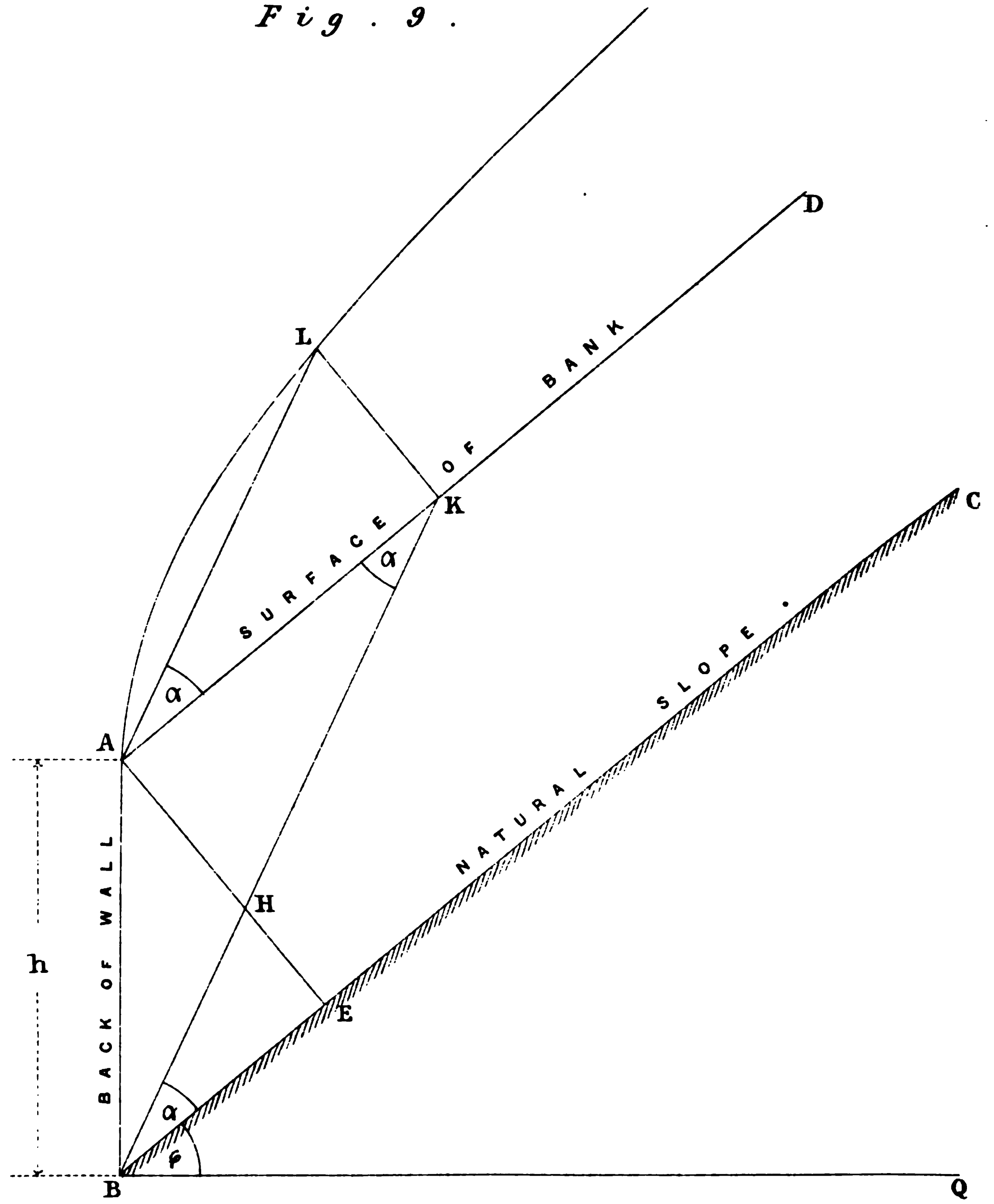
Fig. 9.



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Infinite Surcharge.

If the surface of the ground is parallel to the natural slope and of indefinite extent, the solution of the above problem becomes much simpler, and is as follows:

A B (Fig. 9) is the back of the wall,
B C the natural slope, and
A D the surface of ground.

As in the preceding case, suppose the earth thrust to be produced by a mass A B K.

Through A draw A L parallel to B K, and through K draw K L at right angles to A D.

Let fall perpendicular A E on B C, cutting B K in H.

Now as before,

$$\begin{aligned} \text{Earth thrust produced by prism A B K} & \\ &= \text{weight of A B K tan. } \alpha \\ &= \tfrac{1}{2}\, w_1 \text{A E} \times \text{A K tan. } \alpha \\ &= \tfrac{1}{2}\, w_1 \text{L K . A E.} \end{aligned}$$

$$\text{For angle L A K} = \text{A K B} = \text{K B C} = \alpha$$

$$\begin{aligned} \text{Earth thrust} &= \tfrac{1}{2}\, w_1 \text{L K . A E} \\ &= \tfrac{1}{2}\, w_1 \text{A H . A E.} \end{aligned}$$

Where A E is a constant and A H a varying quantity representing the thrust for different slopes of B K,

A H will be greatest when B K is parallel to A D, in which case A H becomes equal to A E;

therefore

$$\text{Maximum earth thrust} = \tfrac{1}{2}\, w_1 \text{A E}^2.$$

19. Maximum earth thrust $= \frac{1}{2}\, \mathbf{w_1\, h^2 \cos.^2 \phi}$.

This result agrees with that of Neville.

Rankine, who treats the question more generally and abstrusely, makes

20. Maximum earth thrust $= \frac{1}{2} \mathbf{w_1 h^2 \cos. \phi}$,

a higher value than that of Neville. The smaller the value of ϕ the more nearly will the two results agree.

The locus of the point L (Fig. 9) gives a curve, any ordinate to which such as L K, drawn at right angles to A D, represents the amount of earth thrust, produced by any particular prism A B K, acting as one mass.

Problem IV.

To find the resultant pressure produced by the earth thrust and the weight of wall, and to calculate its effect.

A dock wall must be made sufficiently strong to stand when the dock is empty, and therefore, in calculating its stability, the supporting power of the water will be neglected.

A B C D (Fig. 10) represents the cross section of a rectangular wall 1 foot in length.

Let

h = **height of wall**
b = **breadth of wall**
$\mathbf{w_1}$ = **weight of 1 cube foot of earth**
$\mathbf{w_2}$ = „ „ „ **masonry**
q = **ratio of distance between centre of pressure and centre of base to breadth of base**

Draw the triangle of forces thus:

1. Through G (the centre of gravity of wall) draw G Q vertical and from point K (situated at a height F K = $\frac{1}{3} h$ above base) set off K T downwards = $b\,h\,w_2$ to represent *weight of wall.*

2. Draw T S horizontal = $\frac{1}{2} w_1 h^2 \tan.^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$ to represent *thrust of earth.*

3. Join K S, which will give the *resultant pressure* cutting the base in E.

Plate 8.

Fig. 10.

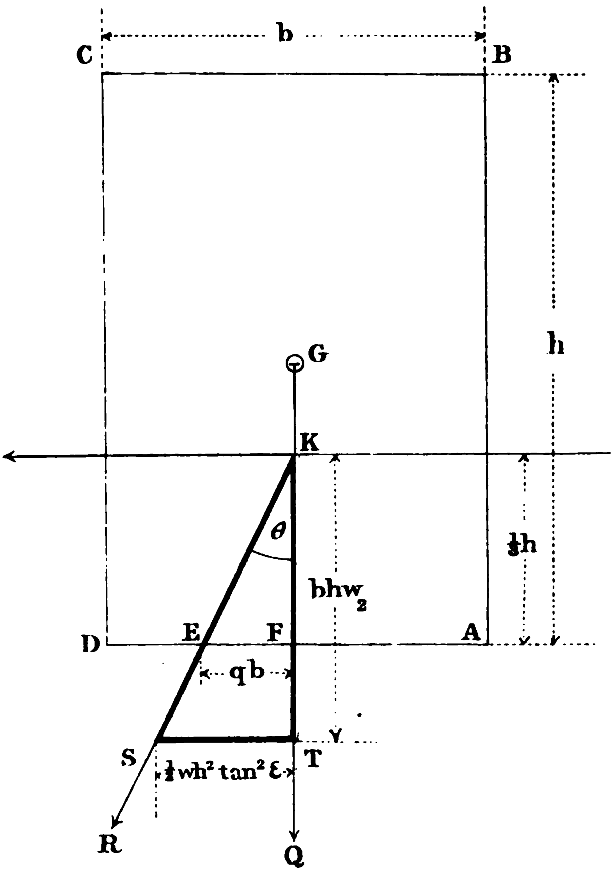
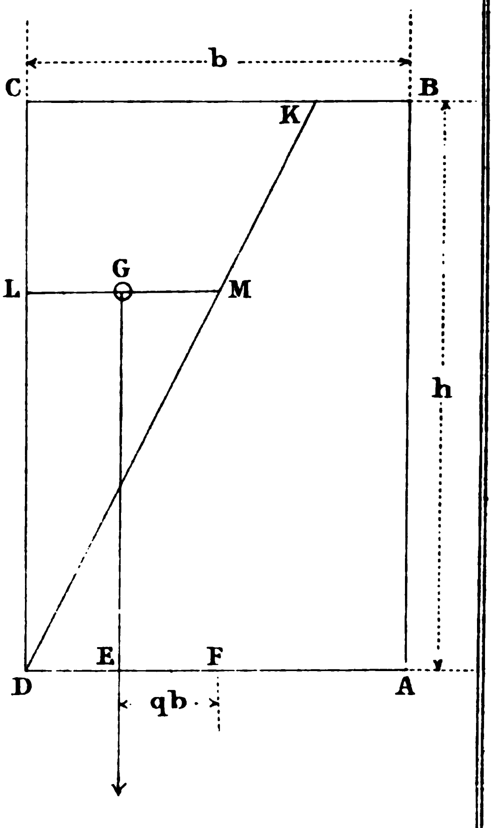


Fig. 11.



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The point E is called the *centre of pressure*, and its distance E F from centre of base $= q\,b$.

Let angle R K Q which resultant makes with vertical $= \theta$, and let $\left(\frac{\pi}{4} - \frac{\phi}{2}\right) = \epsilon$

Now

$$\tan.\ \theta = \tfrac{1}{2}\, w_1\, h^2 \tan.^2 \epsilon \times \frac{1}{b\, h\, w_2}$$

$$21.\quad \tan.\ \theta = \tfrac{1}{2} \cdot \frac{w_1}{w_2} \times \frac{h}{b} \times \tan.^2 \epsilon$$

In order therefore that the wall may not slide forwards, the angle θ must not exceed ϕ_1 (the limiting angle of friction of base of wall on its foundation),

$$22.\quad \text{or } \tan\ \phi_1 \text{ must be less than } \tfrac{1}{2}\, \frac{w_1\, h}{w_2\, b} \times \tan.^2 \epsilon$$

In order to find the ratio of h to b when the wall is just on the point of sliding, the following equation is deduced from Equation 21 :

$$23.\quad \frac{h}{b} = \frac{w_2}{w_1} \times 2 \tan.\ \phi_1 \cot.^2 \epsilon$$

Assuming that ratio of weight of masonry to that of earth $= \frac{5}{4}$ which is a very fair average value, then

$$24.\quad \frac{h}{b} = 2{\cdot}5 \tan.\ \phi_1 \cot.^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$

This equation determines the frictional stability of the wall: it is now necessary to find the relation of h to b so that q may not exceed the safe limit assigned to it by practical engineers.

If q is greater than $\frac{1}{2}$, the resultant will fall without the base, and the wall will consequently overturn.

Before this can possibly take place however, the materials of the face of the wall, or the front portion of the foundations, will have

given way by being crushed; it becomes essential therefore to give q some smaller value to ensure the safety of the structure.

$$25. \begin{cases} \textbf{English engineers make } q = \tfrac{1}{4} \\ \textbf{French} \quad ,, \quad ,, \quad q = \tfrac{3}{8} \end{cases}$$

Referring again to Fig. 10,

$$\tan.\,\theta = \frac{EF}{FK} = \frac{3qb}{h}$$

∴ from Equation 21

$$\frac{3qb}{h} = \tfrac{1}{2} \cdot \frac{w_1 h}{w_2 b} \times \tan.^2 \epsilon$$

or

$$q = \tfrac{1}{6} \cdot \frac{w_1 h^2}{w_2 b^2} \times \tan.^2 \epsilon$$

$$26. \quad \frac{h}{b} = \cot.\,\epsilon \sqrt{6\,q \times \frac{w_2}{w_1}}$$

Assume as before that $\frac{w_2}{w_1} = \frac{5}{4}$,

therefore when $q = \frac{1}{4}$

$$27. \quad \frac{h}{b} = 1\cdot369 \cot.\,\epsilon$$

and when $q = \frac{3}{8}$

$$28. \quad \frac{h}{b} = 1\cdot677 \cot.\,\epsilon$$

The table below gives the ratios of heights to breadths of walls of rectangular profile for different angles of repose of earth supported; These ratios are deduced from Formulæ 27 and 28.

TABLE II.

ϕ	$\frac{h}{b}$ when $q = \frac{1}{4}$.	$\frac{h}{b}$ when $q = \frac{3}{8}$.	ϕ	$\frac{h}{b}$ when $q = \frac{1}{4}$.	$\frac{h}{b}$ when $q = \frac{3}{8}$.
°			°		
0	1·369	1·677	35	2·630	3·222
5	1·494	1·830	40	2·936	3·596
10	1·631	1·998	45	3·305	4·049
15	1·792	2·185	50	3·761	4·607
20	1·955	2·395	55	4·342	5·319
25	2·149	2·632	60	5·109	6·258
30	2·371	2·905			

It will be observed from inspection of the above table that, when the friction between the particles of earth supported, is entirely destroyed, by saturation from water (the result of bad drainage, or any other cause),

ϕ will equal nothing, in which case it will be necessary to make

$$\text{Breadth of base of wall} = \tfrac{3}{4} \text{ height}$$

in order that q may not exceed $\frac{1}{4}$.

By filling in behind the wall with care and paying attention to the drainage, the amount of the earth thrust may be very much reduced, and for all ordinary cases it will be quite sufficient to make

$$\text{Breadth of base of wall} = \tfrac{1}{3} \text{ height,}$$

which will enable the structure to withstand a pressure of earth whose

$$\text{Angle of repose} = 42^\circ,$$

and at the same time keep q within the safe limit of $\frac{1}{4}$, which will also almost always ensure the frictional stability as well as the resistance to crushing and overturning.

The conditions of stability of a wall of rectangular section have now been fully investigated. This is not however the most economical form of profile as regards the amount of material employed, since it is possible, as will be seen by Problem V., to give the face of the wall a considerable batter without altering its stability.

Problem V.

To transform a rectangular profile into one of equal strength, having a straight sloping face.

A B C D (Fig. 11) represents the cross section of a rectangular profile.

The resultant pressure cuts the base A D in E.

In C D take point L so that C L = $\frac{1}{3}$ height of wall. Through

L draw L M horizontal. Through E draw E G vertical, cutting L M in G. Make G M = L G. Join D M and produce it to meet C B in K.

Then the profile A B K D will be of the same strength as the rectangular profile A B C D.

For since the centre of gravity G of triangle C D K is situated vertically over the centre of pressure E, the removal of the portion C D K can take place without affecting the stability of the wall.

Let

$$\left.\begin{array}{lcl} \textbf{Breadth of wall A D} \ldots & = & \mathbf{b} \\ \textbf{Height of wall A B} \ldots & = & \mathbf{h} \\ \textbf{Deviation of centre of pressure from centre of base, E F} \ldots & = & \mathbf{q\,b} \end{array}\right\}$$

Now since

$$\begin{aligned} \mathrm{C\,D} &= \tfrac{3}{2}\,\mathrm{D\,L} \\ \therefore\quad \mathrm{C\,K} &= \tfrac{3}{2}\,\mathrm{L\,M} \\ &= 3\,\mathrm{D\,E} \\ &= 3\,b\,(\tfrac{1}{2} - q)\,; \end{aligned}$$

therefore

29. Breadth K B of new section at top = b (3 q − ½).

If $q = \frac{1}{4}$,

30. { **Breadth of wall at top may be made = ¼ breadth of base without altering stability of wall, in which case the batter of face will be ¾ b to h.**

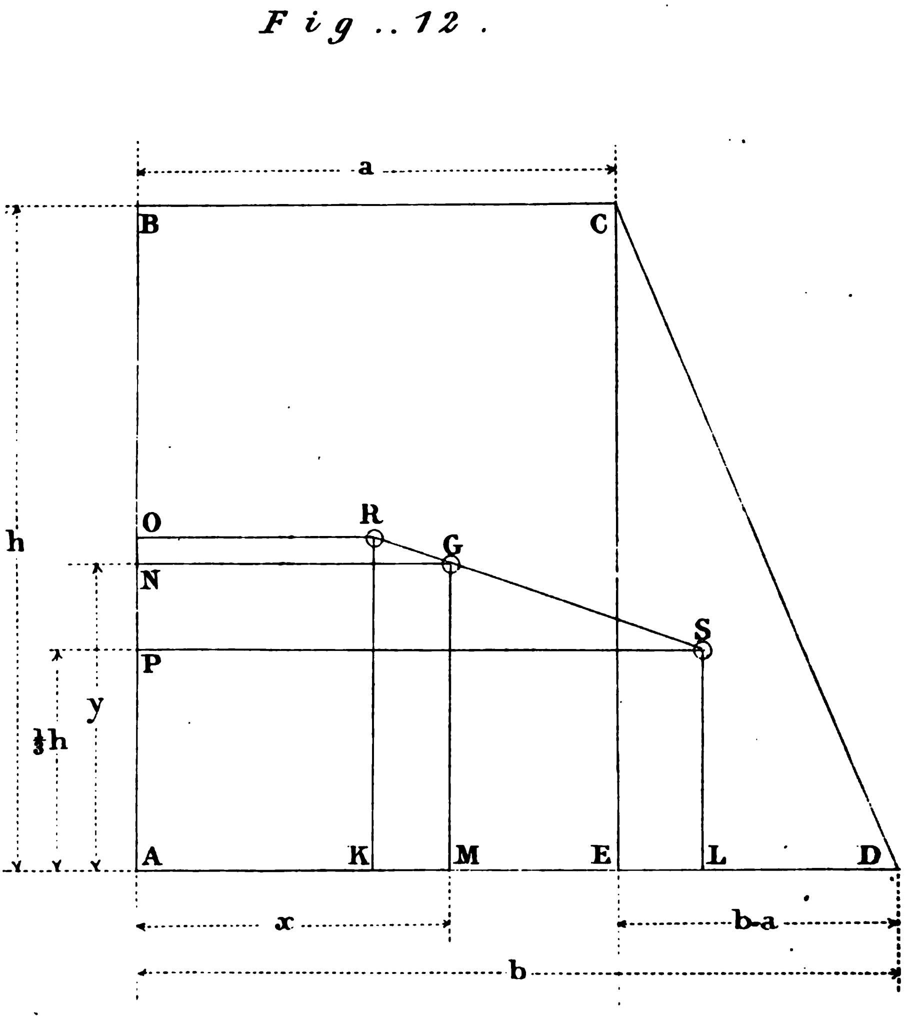
If $q = \frac{3}{8}$,

31. { **Breadth of wall at top may be made = ⅝ breadth of base without altering stability of wall, in which case the batter of face will be ⅜ b to h.**

Problem VI.

To find the position of the centre of gravity of the section of a wall, having a vertical back and a straight battering face.

A B C D (Fig. 12) represents the profile of a trapezoidal wall with a vertical back.



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Let

$$\left.\begin{array}{lll} \textbf{Breadth of wall at top} \ \ldots\ \ldots\ \ldots & \mathbf{BC} & = \mathbf{a} \\ \quad ,, \quad ,, \quad \textbf{bottom} \ \ldots\ \ldots & \mathbf{AD} & = \mathbf{b} \\ \textbf{Height of wall} \ \ldots\ \ldots\ \ldots\ \ldots\ \ldots & \mathbf{AB} & = \mathbf{h} \end{array}\right\}$$

Through C draw C E vertical, cutting base in E.

Find centre of gravity of rectangle B E by drawing O R, K R through O and K, the middle points of A B, A E to intersect in R.

Find centre of gravity of triangle C E D by drawing P S through P where $AP = \frac{1}{3} AB$ and L S through L where $EL = \frac{1}{3} ED$ to intersect in S.

Join R S.

The intersection of the line joining the middle points of B C, A D will give G the centre of gravity of figure A B C D.

Draw ordinates through R, G, and S′ at right angles to A D and A B respectively.

Let

$$\left.\begin{array}{ll} \textbf{Ordinate } \mathbf{GN} & = \mathbf{x} \\ \quad ,, \qquad \mathbf{GM} & = \mathbf{y} \end{array}\right\}$$

It is required to find the values of x and y in terms of a, b, and h.

Now

$$\frac{KM}{ML} = \text{area } ECD : \text{area } ABCE$$

$$= \frac{b - a}{2a}$$

and

$$KM = x - \tfrac{1}{2} a$$

$$ML = b - x - \tfrac{2}{3}(b - a)$$

So

$$\tfrac{3}{2} \times \frac{2x - a}{b + 2a - 3x} = \frac{b - a}{2a}$$

$$\textbf{32.} \quad \mathbf{x} = \frac{\mathbf{b\,(a + b) + a^2}}{\mathbf{3\,(a + b)}}$$

$$\textbf{33.} \quad \mathbf{DM} = \frac{\mathbf{2\,b\,(a + b) - a^2}}{\mathbf{3\,(a + b)}}$$

Again,

$$\frac{ON}{NP} = \text{area } ECD : \text{area } ABCE$$

$$= \frac{b-a}{2a}$$

and

$$ON = \tfrac{1}{3}h - y$$

$$NP = y - \tfrac{1}{3}h$$

So

$$\tfrac{2}{3} \cdot \frac{h - 2y}{3y - h} = \frac{b-a}{2a}$$

$$34. \quad y = \tfrac{1}{3}h \cdot \frac{2a+b}{a+b}$$

$$35. \quad BN = \tfrac{1}{3}h \cdot \frac{a+2b}{a+b}$$

Problem VII.

To find the alteration in position of centre of gravity of a wall produced by stepping at the back.

A B C D (Fig. 13) represents the profile of a wall with a vertical face and stepped at the back, as is usual, with a view to economizing material.

Through A draw A K vertical and produce C B to meet it in K.

Now the mass of earth A K B between the vertical A K and the back of the wall is supported on the stepping, and thus adds to the stability of the structure.

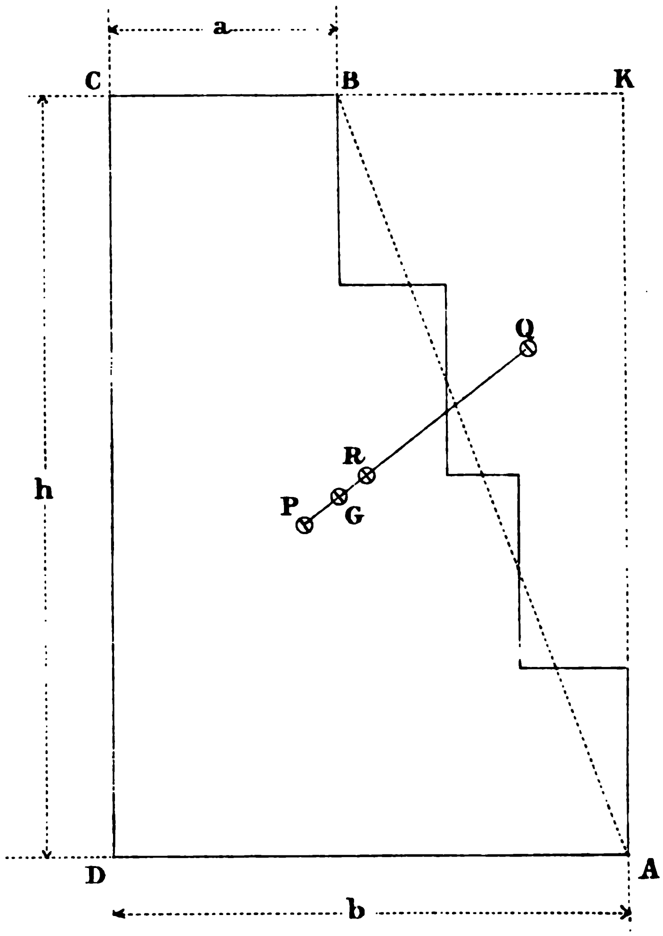
The effect is exactly the same as if the wall was of rectangular section A K C D, the portion A K B being built of a lighter material of specific gravity equal to that of earth.

Join A B, which will give a triangle A K B = mass of earth resting on stepping, near enough for all practical purposes.

It is required to find the position of the centre of gravity of the composite mass A K C D.

Determine P the centre of gravity of A B C D and Q the centre of gravity of A K B by method shown in preceding problem.

Fig. 13.



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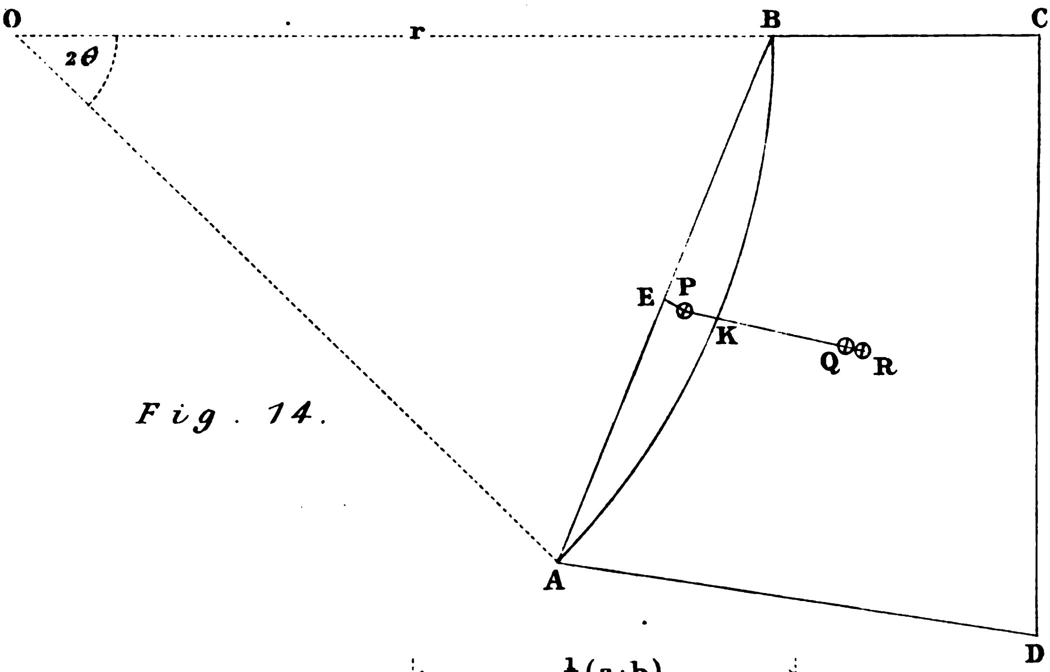

Fig. 14.

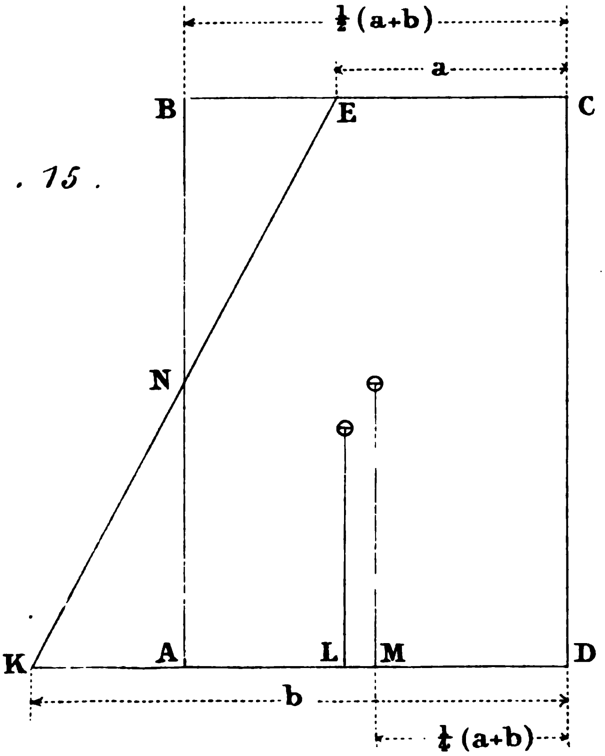

Fig. 15.

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Join P Q.

Let point G in P Q be required centre of gravity of composite mass.

Let

Weight of 1 cubic foot of earth		=	w_1	}
,, ,, ,, **masonry**		=	w_2	
Breadth of wall at top	**B C**	=	**a**	
,, ,, **base**	**A D**	=	**b**	
Height of wall	**D C**	=	**h**	

Now

$$P G \times \tfrac{1}{2} w_2 h \,.\, (a + b) = Q G \times \tfrac{1}{2} w_1 h \,.\, (b - a)$$

$$36. \quad \frac{P G}{Q G} = \frac{b - a}{b + a} \cdot \frac{w_1}{w_2}$$

The intersection R of diagonal A C with P Q gives the centre of gravity of area A K C D.

The point G therefore must always lie between its extreme limits P and R.

If the weight of masonry = weight of earth, stepping at the back will not affect the stability of the wall in any way whatsoever.

In most cases the difference between the weight of earth and masonry is so small, that the ordinary margin for safety allowed for in the calculation of the stability of a rectangular section will include the reduction in the moment of the wall caused by making steps at the back.

Problem VIII.

To find the alteration in position of the centre of gravity of a wall, produced by giving its face a curved batter instead of a straight one.

A B C D (Fig. 14) represents the profile of a wall, with a straight battering face A B.

The arc A K B, whose centre is at O, is the corresponding circular batter, passing through the points A and B.

E

Bisect A B in E and draw E P at right angles to A B.

Let P be the centre of gravity of segment A K B, and Q the centre of gravity of A B C D.

The position of P is determined by the following formula. (See Parkinson's *Mechanics*, p. 103.)

$$37.\quad \mathbf{E\,P} = \tfrac{2}{3}\,\mathbf{r} \cdot \frac{\mathbf{sin.}^3\,\theta}{\theta - \mathbf{sin.}\,\theta\,\mathbf{cos.}\,\theta} - \mathbf{r\ cos.}\,\theta$$

Also

38. Weight of segment A K B $= \mathbf{w}_2\,\mathbf{r}^2\,(\theta - \mathbf{sin.}\,\theta\,\mathbf{cos.}\,\theta)$

Where

Radius O B	=	$\mathbf{r}$
Angle A O B	=	2θ
$\dfrac{\textbf{Arc A K B}}{\mathbf{2\,r}}$	=	θ
Weight of 1 cubic foot of masonry	=	$\mathbf{w}^2$

The position of Q is determined by Problem VI.

Join P Q and produce it to R.

Now if R be the centre of gravity of A K B C D,

$$\text{Q R} \times (\text{weight of A B C D} - \text{weight of segment A K B})$$
$$= \text{P Q} \times \text{weight of segment A K B.}$$

$$39.\quad \mathbf{Q\,R} = \mathbf{P\,Q} \times \frac{\textbf{weight of A K B}}{\textbf{weight of A B C D} - \textbf{weight of A K B}}$$

This formula gives the position of the centre of gravity R of the profile A K B C D, having a face with a circular curved batter.

If the curve of batter be not a circular arc, the problem becomes much more intricate, and must be solved by an approximate method.

The effect of giving a curved batter to a wall instead of a straight one is to move the centre of gravity of a section farther back from the face, and thus increase the moment of stability of masonry.

PROBLEM IX.

To compare the stability of a wall of rectangular section, with that of one of the same sectional area, but having a straight sloping face.

A B C D (Fig. 15) represents a rectangular profile.

Bisect A B in N.

Now any line, drawn through N, such as K N E cutting B C in E and D A produced in K, will give the face of a trapezoidal section D K E C of the same area as A B C D.

Let M be the point vertically under the centre of gravity of rectangle A B C D, and L the point vertically under the centre of gravity of D K E C.

Now

$$\text{Stability of rectangle B D : stability of trapezium E D} = \frac{MA}{LK}$$

Let

$$EC = a$$

$$KD = b$$

Here

$$AD = \tfrac{1}{2}(a + b)$$

$$MA = \tfrac{1}{4}(a + b)$$

$$LK = \frac{2b(a+b) - a^2}{3(a+b)} \quad \text{(See Equation 33)}$$

$$40. \quad \frac{MA}{LK} = \frac{3}{4} \times \frac{(a+b)^2}{2b(a+b) - a^2}$$

When figure D K E C becomes a triangle, $a = 0$.

41. **Stability of a rectangular profile : stability of triangular profile of same sectional area and height = $\frac{3}{8}$.**

Problem X.

To show how an uniformly varying pressure may be represented graphically by either a triangle or a trapezium; the *centre of pressure* lying vertically under the *centre of gravity* of figure.

Definition.—An uniformly varying pressure acting on a plane surface is such that its intensity at any given point is proportional to the perpendicular distance of that point from a given straight line in the plane of surface acted upon. (See Rankine's *Engineering*, p. 163.)

It has been previously shown that, if the resultant pressure produced by the combined effect of the earth thrust and the weight of wall fall beyond the limits of base, the structure will be overturned.

However, before this can possibly take place, the materials composing the face of the wall or the front of the foundations will have given way by being crushed, because any deviation of the *centre of pressure* from the *centre of base* will increase the intensity of the pressure towards the side on which this deviation takes place.

If the *centre of pressure* coincides with the *centre of base*, the pressure is uniformly distributed, and is the same per unit of area over the whole foundation.

When the *centre of pressure* falls on one side of the *centre of base* the pressure increases towards that side, and is proportional to the amount of deviation, or, in other words, the pressure is an uniformly varying one.

The way in which such a pressure acts is investigated as follows:

Supposing a series of rectangular solids (Fig. 16), whose heights increase in arithmetical progression, to be placed side by side on the area A B C D.

Now the pressures produced by the weights of the solids on their bases will also increase in arithmetical progression.

If between A and B there be an infinite number of these solids,

Fig. 16.

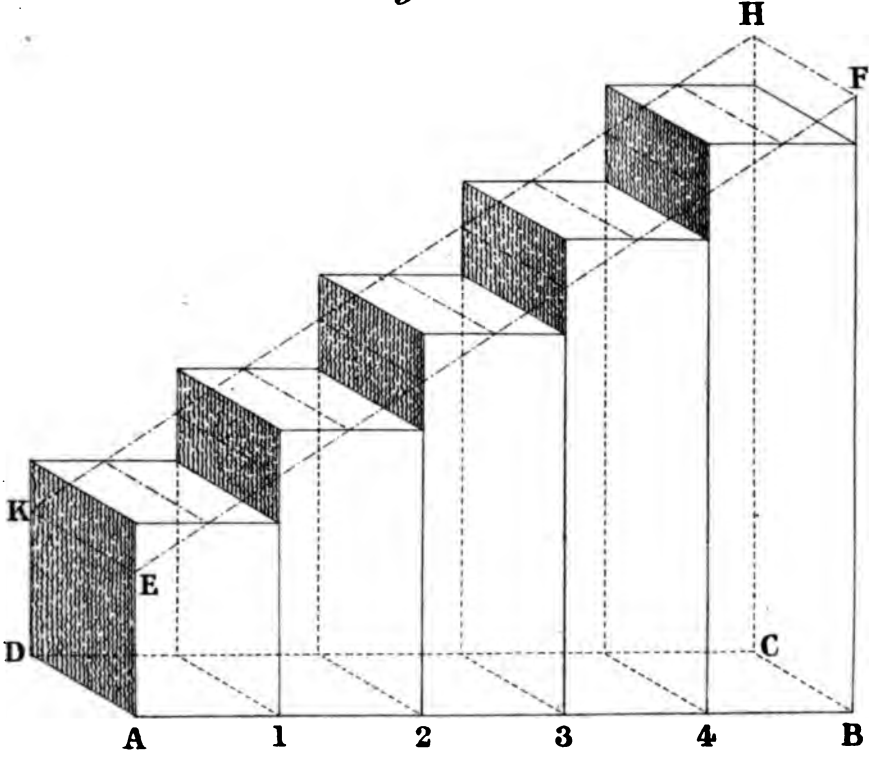

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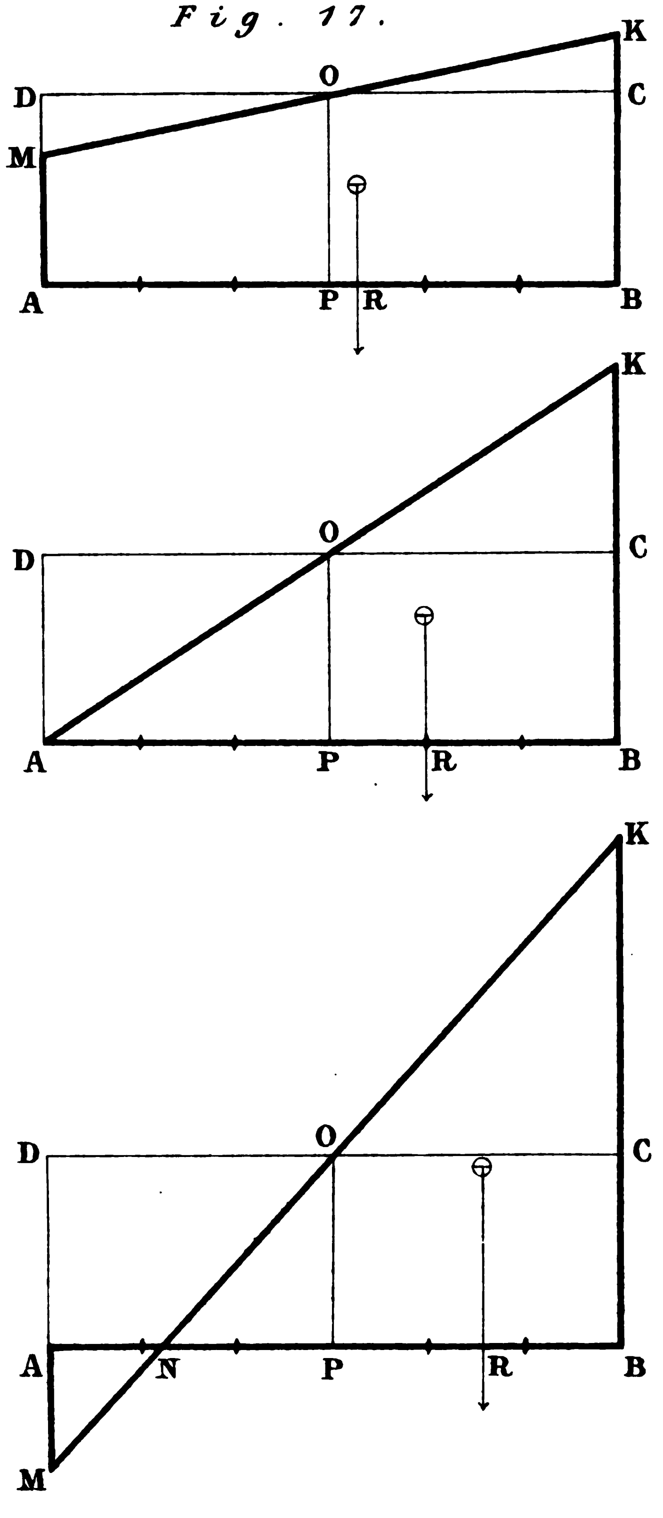
Fig. 17.



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each indefinitely thin, then instead of a series growing larger by perceptible steps, a trapezoidal solid A B H K is obtained, representing their combined effect.

The weight of trapezoid A B H K produces an uniformly increasing pressure on its base A B C D, the total effect of which is equal to weight of unit of volume × area A B F E × D A, and acts through centre of gravity of figure A B H K.

Also the pressure at any point is proportional to the height of the solid at that point.

Thus an uniformly varying pressure acting on a rectangular surface may be graphically represented by the area of a trapezium, and the centre of pressure will lie vertically under the centre of gravity of figure.

Diagram 17 illustrates the different cases of this problem.

1. When the pressure is of uniform intensity over the whole area exposed, the *ideal figure is a rectangle* A B C D, and the centre of pressure, which lies vertically under centre of gravity of A B C D, passes through P the middle point of the base.

2. When the pressure commences from nothing at A and increases uniformly towards B, where maximum pressure = B K, the *ideal figure is a triangle* A B K, and the centre of pressure passes through point R at a distance P R = $\frac{1}{6}$ breadth of base from centre of base.

3. When the pressure at A is proportional to A M and increases uniformly to a maximum B K at B, *the ideal figure is a trapezium* A B K M, and the centre of pressure R falls within the middle third of base.

4. When the pressure at A is a minus quantity or a tension = − A M becoming nothing at N and then increasing to a maximum pressure B K at B, *the ideal figure assumes the form of a double triangle* A M N K B, and the centre of pressure R falls beyond the middle third of the base.

From the above it is evident that given an uniformly varying pressure acting on a rectangular area: if the greatest and least pressures be known, then the point where the centre of pressure cuts

the base may be determined, by drawing the corresponding ideal figure and finding the point in the base vertically under its centre of gravity.

The position of the centre of pressure therefore depends on the amount of the greatest and least pressures.

Also

$$\text{Mean pressure per unit of area} = \tfrac{1}{2} \text{ sum of greatest and least pressures}$$

and

$$\text{Mean pressure per unit of area} = \frac{\text{total pressure}}{\text{area of base}}$$

It now becomes necessary to work out the converse problem, namely, to find the maximum pressure in terms of the mean pressure and of the distance between centre of pressure and centre of base.

PROBLEM XI.

Given an uniformly varying pressure acting on a rectangular surface and the amount of deviation of the centre of pressure from the centre of gravity of that surface. To find the maximum pressure produced in terms of the deviation and of the mean pressure.

Let A B (Fig. 18) be the cross section of a rectangular plane surface acted upon by an uniformly varying pressure.

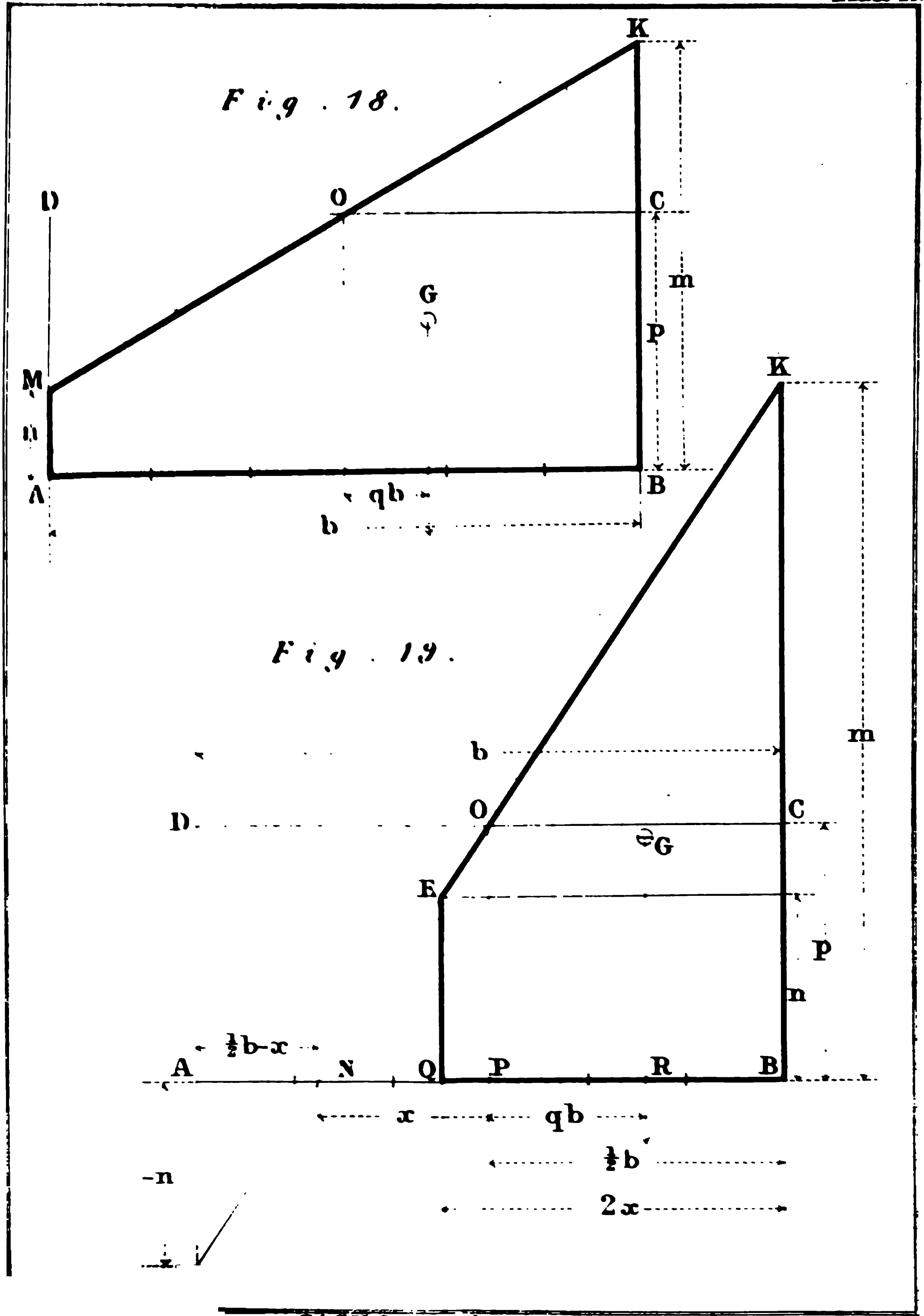
At A draw A M at right angles to A B, to represent the *least pressure.*

At B draw B K at right angles to A B, to represent the *greatest pressure.*

Now trapezium A B K M is the *ideal figure* representing the effect of the uniformly varying pressure, and P O drawn at right angles to A B through middle point P gives the mean pressure.

Find centre of gravity G of A B K M.

Then point R vertically under G is the centre of pressure.



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Let

Maximum pressure	.. **B K**	=	**m**	**unknown**
Minimum pressure	.. **A M**	=	**n**	**quantities.**
Mean pressure	.. **P O**	=	**p**	**known**
Length of	.. **A B**	=	**b**	**quantities.**
Amount of deviation	.. **P R**	=	**q b**	

Now

$$p = \text{mean intensity of pressure per unit of surface}$$

$$= \frac{\text{total amount of vertical forces acting on surface A B}}{\text{area of surface A B}}$$

Also

$$p = \tfrac{1}{2} \text{ sum of greatest and least pressures.}$$

The following equations enable m to be found in terms of p and q:

$$\left.\begin{aligned} RB &= \tfrac{1}{3} b \left(\frac{m + 2n}{m + n}\right) \\ RB &= \tfrac{1}{2} b - q b \\ p &= \tfrac{1}{2} (m + n) \end{aligned}\right\} \text{(See Problem VI., Equation 34)}$$

Therefore

$$b\left(\tfrac{1}{2} - q\right) = \tfrac{1}{3} b \left(\frac{4p - m}{2p}\right)$$

Solving which,

$$\textbf{42.} \quad \mathbf{m = p\,(1 + 6\,q)}$$

By this equation all cases where q is less than $\frac{1}{6}$ may be determined.

If $q = \frac{1}{6}$ then the ideal figure is a triangle, and

$$\textbf{43.} \quad \mathbf{m = 2\,p}$$

When q is greater than $\frac{1}{6}$ the problem is solved thus:

The diagram and notation will be exactly similar to that used in the preceding case except that A M being a tension is drawn downwards instead of upwards.

Let N (Fig. 19) be the point where K M cuts A B. Make

N Q = A N, and draw Q E at right angles to A B cutting M K in E.

Now the pressures represented by triangle A M N will be just neutralized by pressures represented by triangle N Q E.

So that B Q E K will be the *ideal figure* representing the total effect of the forces acting on A B, and the centre of pressure R will lie vertically under G, the centre of gravity of B Q E K.

P is the middle point of A B.

Let

$$PN = x$$

$$QP = AP - 2AN \quad \text{and} \quad AN = \tfrac{1}{2}b - x$$

$$QP = \tfrac{1}{2}b - b + 2x$$

$$= 2x - \tfrac{1}{2}b$$

And

$$QB = QP + \tfrac{1}{2}b$$

$$= 2x$$

Again

$$\frac{NP}{PO} = \frac{OC}{CK} \quad \text{and} \quad CK = m - p$$

$$\frac{x}{p} = \tfrac{1}{2}b \times \frac{1}{m-p}$$

$$x = \tfrac{1}{2} \times \frac{bp}{m-p}$$

Therefore

$$QB = \frac{bp}{m-}$$

Also

$$p = \tfrac{1}{2}(m-n)$$

$$n = m - 2p$$

Now

$$RB = \tfrac{1}{2}b - qb$$

Also

$$RB = \tfrac{1}{3}QB \times \frac{m+2n}{m+n} \qquad \text{(See Problem VI., Equation 34)}$$

$$= \tfrac{1}{3} \cdot \frac{bp}{m-p} \times \frac{m+2n}{m+n}$$

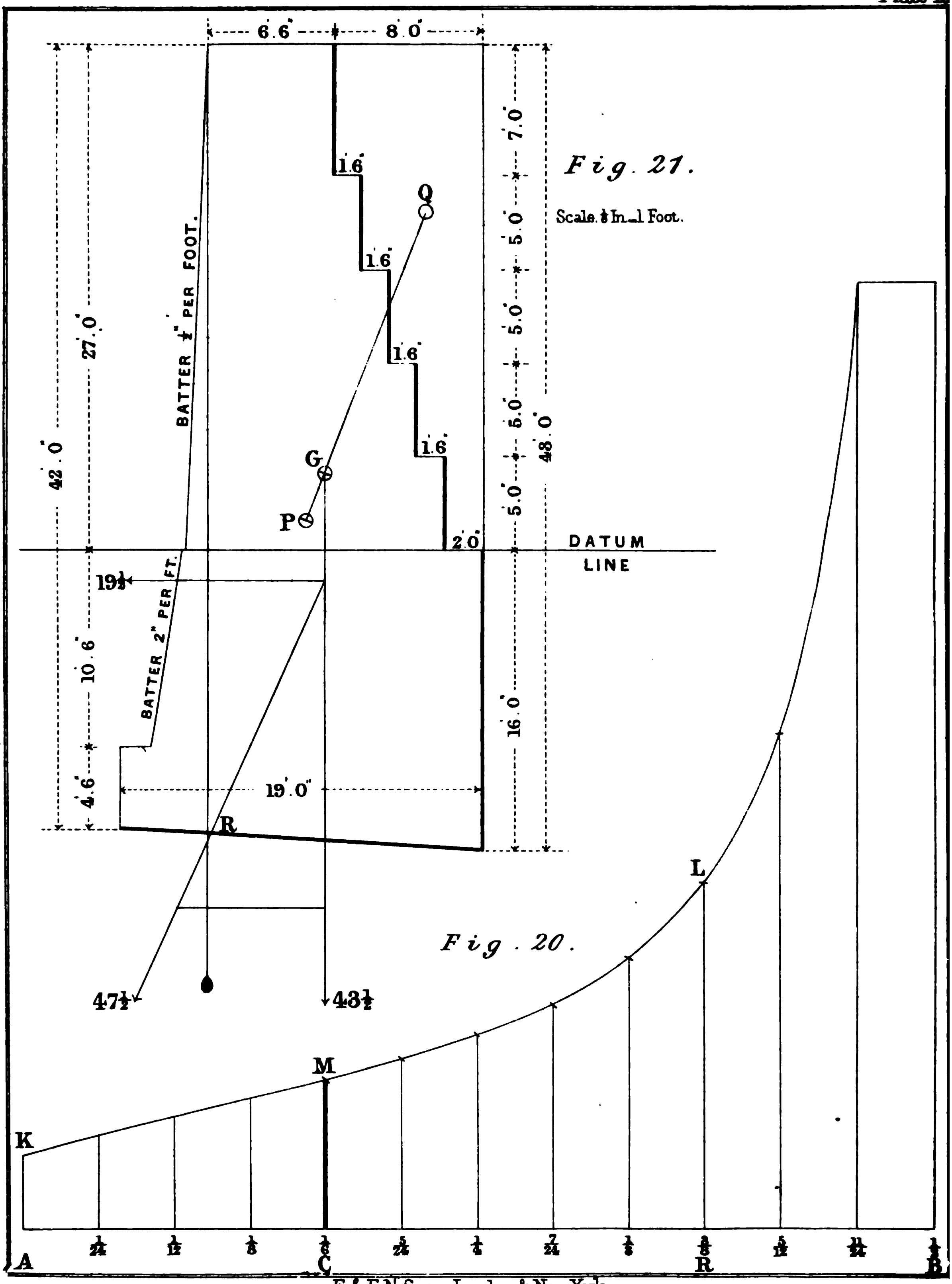

Fig. 21.

Fig. 20.

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And

$$\frac{m + 2n}{m + n} = \tfrac{1}{2} \cdot \frac{3m - 4p}{m - p}$$

So

$$RB = \tfrac{1}{6} \cdot \frac{bp}{m - p} \times \frac{3m - 4p}{m - p}$$

Therefore

$$b\left(\tfrac{1}{2} - q\right) = \tfrac{1}{6} \cdot \frac{bp}{(m - p)^2} \times (3m - 4p)$$

From which equation the following quadratic is obtained:

$$m^2 - m \cdot \frac{3 - 4q}{1 - 2q} \times p = \tfrac{1}{3} p^2 \frac{6q - 7}{1 - 2q}$$

Solving this,

$$44. \quad m = p \cdot \frac{1}{2(1 - 2q)} \left(3 - 4q + \frac{1}{\sqrt{3}} \cdot \sqrt{8q - 1}\right)$$

The table below is deduced from Equations 42, 43, and 44, and gives the coefficient of p for different values of q.

TABLE III.

Values of q.	Coefficients of p.
0	1·000
$\frac{1}{24}$	1·250
$\frac{1}{12}$	1·500
$\frac{1}{8}$	1·750
$\frac{1}{6}$	2·000
$\frac{5}{24}$	2·261
$\frac{1}{4}$	2·577
$\frac{7}{24}$	3·000
$\frac{1}{3}$	3·618
$\frac{3}{8}$	4·632
$\frac{5}{12}$	6·645
$\frac{11}{24}$	12·656
$\frac{1}{2}$	infinite

This table is shown graphically on Fig. 20.

A B representing $\frac{1}{2}$ breadth of rectangular surface acted on by an uniformly varying pressure is 6 inches long.

A K representing the mean pressure is $\frac{1}{2}$ inch long.

Therefore, if A R be the distance between centre of pressure and centre of gravity of surface acted on, then the length of R L measured with a $\frac{1}{2}$ inch scale will give the value of the maximum pressure.

Where the amount of deviation of centre of pressure is less than $\frac{1}{6}$ breadth of base it will be observed that the values of the maximum pressures increase in arithmetical progression and the line K M is straight.

Beyond this, however, they increase much more rapidly, and when centre of pressure falls on outer edge of A B the maximum pressure = infinity.

A table is added below showing the different values of q when the coefficients of p are whole numbers.

Coefficients of p.	Values of q.		
1	0		
2	$\frac{1}{6}$	=	$\frac{1}{6.1^2}$
3	$\frac{7}{24}$	=	$\frac{1+6}{6.2^2}$
4	$\frac{19}{54}$	=	$\frac{1+6+12}{6.3^2}$
5	$\frac{37}{96}$	=	$\frac{1+6+12+18}{6.4^2}$
t	$\frac{1+3(t^2-3t+2)}{6.(t-1)^2}$	=	$\frac{1+(6+12+18+\text{\&c. to }(t-2)\text{ terms}}{6.(t-1)^2}$

SUMMARY.

The object of the treatise and a list of the different authorities on the subject is given at the commencement.

Next follows a catalogue of the Problems connected with dock walls, the general results obtained being these, namely:

A dock wall has to resist a pressure of water acting against its face and a thrust of earth acting against its back.

These two forces counteract each other, so that when the dock is full it is only the preponderance of the greater over the less which tends to endanger the stability of the structure.

During construction, however, and at other times subsequently, it is necessary that the dock should be empty.

The strength of the section of the wall must therefore be sufficient to withstand the thrust of the earth, by means of its weight only, being unassisted by the supporting power of the water.

Pressure of Water.

(*See Problem I.*)

The pressure of water varies as the square of the depth and its

Amount = area of face immersed × depth of centre of gravity of face below surface × weight of 1 cubic foot of water.

Direction is normal to face of profile.

Point of Application is $\frac{2}{3}$ depth of face below surface.

Pressure of Earth.

(*See Problems II. and III.*)

The greatest pressure of earth which can ever be brought to bear against the back of a wall is when both the friction and adhesion between the particles of the soil are destroyed by water or any other cause.

The amount of the pressure in this case is equal to that which

would be produced by a fluid of the same specific gravity as earth, and may be determined by Problem I., substituting w_1 for w in Formula 1.

The only force which is likely, except under extraordinary circumstances, to come into play is that of earth possessing a certain amount of friction between its particles, which varies according to the angle of repose.

Adhesion is neglected, being too uncertain a quantity to be dealt with.

Taking friction into account, the maximum earth thrust is equal to that which would be produced by a fluid, the weight of each cubic foot of which is $w_1 \tan.^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$, and may be determined by Problem I., substituting $w_1 \tan.^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$ in Formula 2.

The wedge of earth to which this thrust is due is formed between the back of the wall and the plane bisecting the complement of the angle of repose.

The above only holds good when the surface of the ground is horizontal and the back of the wall vertical.

When this is not so, the wedge of maximum earth thrust is equal in sectional area to the triangle formed by letting fall a perpendicular on the natural slope from the point where plane of rupture cuts the surface of ground.

These more complicated cases and also that of surcharge are fully discussed in the notes to Problem III.

Table I. gives the amount of earth thrust against the vertical back of a wall for different values of ϕ, the angle of repose.

Resultant Pressure.

(*See Problem IV.*)

The pressures which exist against a wall have now been determined, and it is next shown in what manner they tend towards causing failure.

These forces may combine to destroy the wall in any or all of the following ways, namely:

1. Causing the structure to heel over or overturn.
2. Causing it to slide bodily forwards.
3. Crushing the materials of which the wall is constructed.

The point where the resultant pressure produced by the earth thrust and the weight of wall is called the *centre of pressure.*

It is found by practice that in order to avoid failure by any of the above-mentioned causes, the *centre of pressure* should not deviate from the *centre of gravity* of the area of base by a distance exceeding a certain fraction of the whole breadth of base.

This fraction is called q, and is made by English engineers, $\frac{1}{4}$; French engineers, $\frac{3}{8}$.

The value of q being fixed, the ratio of the *height* of a wall of rectangular section to its *breadth* may be found by Formula 26.

The relation of height of wall to breadth when it is on the point of sliding forwards is given by Formula 23.

Table II. gives the various ratios of heights to breadths for different values of angle of repose of earth.

Transformation of Profile.

(*See Problem V.*)

A rectangular section is the least economical form in which a wall can be built, with regard to the amount of material employed.

The method of converting a rectangular profile into one of equal strength with a straight battering face is fully explained in Problem V., the general principle being that,

As long as the breadth of base remains unaltered, any portion of the front of the wall may be removed without altering its stability, provided that the point vertically under the centre of gravity of the portion taken away falls between the centre of pressure and the face.

The Centre of Gravity of a Trapezoidal Profile.

(*See Problem VI.*)

Stepping at Back of Wall.

(*See Problem VII.*)

The effect of stepping the masonry behind a wall is investigated in this Problem, the result being that, in proportion as the weight of earth is less than that of masonry, the centre of gravity of profile is brought nearer face of wall.

Curved Batter.

(*See Problem VIII.*)

The effect of replacing a straight batter for the face of a wall by a curved one is to add to the stability of the structure by bringing the centre of gravity of profile nearer back of wall.

Stability of Trapezoidal Profile.

(*See Problem IX.*)

The comparative stability of a rectangular section and one with a straight battering face but of same area is given by Formula 40.

The strength of a rectangular profile is to that of a triangular one of same height and sectional area as 3 to 8.

Uniformly Varying Pressure.

(*See Problem X.*)

It is here shown that an uniformly varying pressure may be graphically represented by any of the following plane figures:

1. A trapezium.
2. A triangle.
3. A double triangle.

The centre of pressure lies vertically under the centre of gravity of the ideal figure.

CRUSHING POWER OF EARTH THRUST.

(*See Problem XI.*)

This Problem shows how the graphical representation of an uniformly varying pressure may be applied to determining the maximum power of the resultant force, produced by earth thrust and weight of wall, to crush the materials of which the face of wall or foundations are composed.

The mean pressure per unit of area exposed and the value of q being known, then

Formulæ 42, 43, and 44 give the value of the maximum force, which must not exceed the resistance of materials of which wall is composed, to crushing.

Table III. and Diagram 20 show the amount of maximum pressure for different values of q.

PROCESS OF DESIGNING THE SECTION OF A DOCK WALL DEDUCED FROM THE PRECEDING PROBLEMS.

In determining the proportions and outline of a profile, the following leading principles must be borne in mind.

The wall must be strong enough to stand when the dock is empty.

The amount of the earth thrust behind wall depends on the adhesion and friction between the grains of earth.

Table I. and accompanying diagram illustrate most forcibly how rapidly the pressure of earth increases as the friction between its particles diminishes.

Now if sufficient care be taken with the method of filling in behind the wall and the drainage, it is possible to prevent the angle of repose, on which the amount of friction depends, from decreasing beyond a certain limit.

In this way considerable economy of design may be effected.

Assuming the filling to be well put in, there never need be any danger of the angle of repose becoming less than 30°.

30° may therefore in all ordinary cases be taken as a safe value for ϕ.

If $\phi = 30°$,

The earth thrust $= \frac{1}{3}$ the greatest pressure which can ever act against the back of wall; namely, that of a fluid of same specific gravity as earth. (See Table I.)

Or

$$\text{Earth thrust} = \tfrac{1}{6} w_1 h^2$$

where

$$\left.\begin{array}{l} w_1 = \text{weight of 1 cubic foot of earth} \\ h = \text{height of wall} \end{array}\right\}.$$

And referring to Table II. it will be seen that in order to resist this thrust and yet keep q within the safe limit of $\frac{1}{4}$,

Breadth of wall should be $\frac{1}{2}$ height.

The amount of earth thrust being known, then, in order to ensure

1. Stability of position of wall,

Resultant pressure produced by earth thrust and weight of wall should not cut the base at a greater distance from its centre than $\frac{1}{4}$ of its whole breadth.

2. Frictional stability.

The angle at which the resultant cuts base should be as nearly at right angles as possible, and should under no circumstances be as small as $(90° - \phi_1)$,

ϕ_1 being the limiting angle of friction between the surface of base of wall and the foundation on which it rests.

3. The materials of wall not being crushed.

The maximum pressure determined by Formulæ 42, 43, and 44, should always be less than the resistance of the materials of the wall or foundations to crushing.

The process of designing according to the above principles is as follows.

Fix upon a safe value for the angle of repose of earth supported, so that by careful filling and draining behind wall this limit need never be reached.

The height of wall, being a quantity already settled by the practical requirements of the case, the breadth of base necessary for a rectangular profile, of the given height, and supporting a pressure of earth whose angle of repose is known, may be found by Table II.

This rectangular section may now be transformed into a more economical one, without endangering its stability, by having a battered face in front and stepping behind, as long as the limits stated in Problems V. and VII. are not exceeded.

Give the base and courses of masonry a sufficient amount of rake backwards to ensure the frictional stability of the wall.

Practical Example.

The manner in which the foregoing principles are applied in practice is shown on Fig. 21, which represents the section of the north wall of the Canada Half-tide Dock, Liverpool, executed under the able superintendence of Mr. G. F. Lyster, the Engineer-in-chief. The dimensions were taken during construction by the author, and are as follows:

	ft.	in.
Height	43	0
Breadth of base	19	0
Breadth at top	6	6
	sq. ft.	
Sectional area of wall above datum	266	
„ „ „ below „	270	
Total sectional area of masonry	536	
	sq. ft.	
Sectional area of earth supported on stepping behind wall ..	141	
	lbs.	
Weight of masonry per foot forward at 150 lbs. to the cubic foot	80,400	
Weight of earth per foot forward at 120 lbs. to the cubic foot	16,920	
Total pressure on foundations	97,320 lbs., or 43·4 tons.	

It appears therefore that the weight per foot forward of the masonry and the earth supported on stepping (which adds to the stability of wall) is nearly $43\frac{1}{2}$ tons.

This pressure acts vertically downwards through the centre of gravity G of the composite mass of earth and masonry.

The thrust of earth behind wall acts horizontally at $\frac{2}{3}$ depth of wall.

The resultant (the combined effect of these two forces) is shown cutting the base at a point $\frac{1}{4}$ breadth of base from centre of base: this being the safe limit assigned by English engineers to distance between centre of pressure and centre of base.

This being the case, if the triangle of forces be drawn, it will be found that amount of

$$\text{Earth thrust per foot forward of wall} = 19\tfrac{1}{2} \text{ tons.}$$

This is the thrust which would be produced by earth having an angle of repose of 26°. For from Formula 8, Problem III.,

$$\tan.\left(45^\circ - \frac{\phi}{2}\right) = \frac{1}{h}\sqrt{\frac{2\,M}{w_1}}$$

where

$$\left.\begin{aligned} M &= 19\tfrac{1}{2} \text{ tons} \\ w_1 &= 120 \text{ lbs.} \\ h &= 43 \text{ feet} \\ \phi &= \text{angle of repose} \end{aligned}\right\}$$

$$\tan.\left(45^\circ - \frac{\phi}{2}\right) = \tfrac{1}{43}\sqrt{39 \times \tfrac{56}{3}}$$

$$= \cdot 627$$

$$\phi = 26^\circ$$

The angle which the resultant pressure makes with the vertical is 21°, which gives the limiting angle of friction of base of wall on its foundation.

Again, since

$$\text{Superficial area of foundation per foot forward} = 19 \text{ square feet}$$

and

$$\text{Total pressure on foundation} = 43\cdot 4 \text{ tons,}$$

therefore

Mean pressure per square foot of foundation = 2·28 tons.

And from Formula 44, and Table III., when deviation of *centre of pressure* from *centre of base* = $\frac{1}{4}$ breadth of base,

Maximum pressure produced = 2·57 mean pressure
= 5·86 tons per square foot.

The results arrived at therefore are that the wall in question is capable of sustaining a pressure of earth whose angle of repose = 26° without the materials of the wall of foundation being subjected to a greater crushing force than 5·86 tons per square foot.

The resistance of sandstone to crushing is about 300 tons per square foot, so that in this case there is an ample margin for safety.

The wall cannot overturn since the point where the resultant cuts the base falls within it.

The wall will not slide forwards unless the limiting angle of friction of base of wall on foundation exceed 21°.

SUMMARY OF FORMULÆ.

Problem I.

Pressure of Water.

Against a wall with straight battering face,

$$P = \tfrac{1}{2} w h^2 \text{ cosec. } \theta \qquad [1]$$

Against a wall with vertical face,

$$P = \tfrac{1}{2} w h^2 \qquad [2]$$

Depth of centre of pressure below surface,

$$\tfrac{2}{3} h \qquad [3]$$

Direction of pressure normal to face of wall.

Problem II.

Pressure of Earth.

Against a wall with vertical back, neglecting friction,

$$T = \tfrac{1}{2} w_1 h^2 \qquad [4]$$

Depth of centre of pressure below surface,

$$\tfrac{2}{3} h \qquad [5]$$

Direction of pressure horizontal.

Taking friction into account,

$$H = \text{weight of wedge of earth} \times \text{tan. } a \qquad [6]$$

PROBLEM III.

Maximum earth thrust against vertical back of wall, taking friction into account, surface of ground being level,

$$M = \frac{1}{2} w_1 h^2 \tan.^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right) \qquad [8]$$

Rankine's formula,

$$M = \frac{1}{2} w_1 h^2 \frac{1 - \sin. \phi}{1 + \sin. \phi} \qquad [9]$$

Depth of centre of pressure below surface,

$$\frac{2}{3} h \qquad [10]$$

Direction of pressure horizontal.

Notes on Problem III.

Maximum earth thrust against back of wall, which is not vertical, taking friction into account, surface of ground not being level,

$$M = \frac{1}{2} w_1 x^2 \qquad [13]$$

$$x = b \tan. \beta - \sqrt{b^2 \tan.^2 \beta - a b \tan. \beta} \qquad [14]$$

$$x = c - \sqrt{c (c - a)} \qquad [15]$$

Maximum earth thrust produced by surcharge,

$$M = \frac{1}{2} w_1 h^2 \cos.^2 \phi \qquad [19]$$

Rankine's formula,

$$M = \frac{1}{2} w_1 h^2 \cos. \phi \qquad [20]$$

PROBLEM IV.

Resultant pressure produced by earth thrust and weight of wall : its effect. Equation determining angle which resultant makes with the vertical,

$$\tan. \theta = \frac{1}{2} \cdot \frac{w_1}{w_2} \times \frac{h}{b} \tan.^2 \epsilon \qquad [21]$$

Ratio of height of rectangular profile to breadth of base when wall is on point of sliding forwards,

$$\frac{h}{b} = \frac{w_2}{w_1} \times 2 \tan. \phi_1 \cot.^2 \epsilon \qquad [23]$$

Ratio of height of rectangular profile to breadth of base, when the point where resultant cuts base falls within safe limits so as not to endanger the stability of structure,

$$\frac{h}{b} = \cot. \epsilon \sqrt{6 q \times \frac{w_2}{w_1}} \qquad [26]$$

Ratio of height to breadth when safe limit = $\frac{1}{4}$,

$$\frac{h}{b} = 1 \cdot 369 \cot. \epsilon \qquad [27]$$

Ratio of height to breadth when safe limit = $\frac{3}{8}$,

$$\frac{h}{b} = 1 \cdot 677 \cot. \epsilon \qquad [28]$$

Problem V.

Transformation of Profiles.

Breadth at top of trapezoidal profile of stability equal to that of rectangular profile on same base and of same height

$$= b\,(3 q - \tfrac{1}{2}) \qquad [29]$$

Breadth of transformed profile at top when $q = \frac{1}{4}$,

$$\text{Batter of face,} \quad \left.\begin{array}{c} \tfrac{1}{4} \text{ breadth of base} \\ \tfrac{3}{4}\, b \text{ to } h \end{array}\right\} \qquad [30]$$

Breadth of transformed profile at top when $q = \frac{3}{8}$,

$$\text{Batter of face,} \quad \left.\begin{array}{c} \tfrac{5}{8} \text{ breadth of base} \\ \tfrac{3}{8}\, b \text{ to } h \end{array}\right\} \qquad [31]$$

PROBLEM VI.

To find the co-ordinates of the centre of gravity of a wall of trapezoidal section; taking the back and base of wall as the axes,

$$x = \tfrac{1}{3} \cdot \frac{a^2 + ab + b^2}{a + b} \qquad [32]$$

$$y = \tfrac{1}{3} h \cdot \frac{2a + b}{a + b} \qquad [34]$$

Distance from toe of wall,

$$DM = \tfrac{1}{3} \cdot \frac{2b(a + b) - a^2}{a + b} \qquad [33]$$

Depth below top of wall,

$$BN = \tfrac{1}{3} h \cdot \frac{a + 2b}{a + b} \qquad [35]$$

PROBLEM VII.

Alteration in position of centre of gravity of section of wall, produced by stepping at the back,

$$\frac{PG}{QG} = \frac{b - a}{b + a} \times \frac{w_1}{w_2} \qquad [36]$$

where

P is centre of gravity of wall stepped behind,
Q is centre of gravity of earth resting on stepping,
G is required centre of gravity of composite mass.

PROBLEM VIII.

Centre of gravity of section of wall with a curved batter on face.
Distance of centre of gravity of circular segment from chord line,

$$= \tfrac{2}{3} r \frac{\sin.^3 \theta}{\theta - \sin. \theta \cos. \theta} - r \cos. \theta \qquad [37]$$

Weight of circular segment,

$$= w_2 r^2 (\theta - \sin. \theta \cos. \theta) \qquad [38]$$

$$QR = PQ \times \frac{\text{weight of circular segment}}{\text{weight of section with curved face}} \qquad [39]$$

where

P is centre of gravity of circular segment,
Q is centre of gravity of trapezoidal section,
R is required centre of gravity of section with curved face.

Problem IX.

The stability of a rectangular profile is to the stability of a triangular profile of same height and sectional area in the proportion of

$$3 : 8 \qquad [41]$$

Problems X. and XI.

Maximum crushing power exerted by an uniformly varying pressure acting on a rectangular surface,

$$p = \frac{\text{total amount of vertical forces}}{\text{area on which they act}}$$

When uniformly varying pressure can be represented graphically by

a trapezium,

$$m = p\,(1 + 6\,q) \qquad [42]$$

a triangle,

$$m = 2\,p \qquad [43]$$

two triangles,

$$m = p \times \frac{1}{2\,(1 - 2\,q)} \left\{3 - 4\,q + \frac{1}{\sqrt{3}} \sqrt{8\,q - 1}\right\} \qquad [44]$$

a rectangle,

$$m = p$$

Symbols made Use of.

Problem I.

P = total water pressure per foot forward of wall.
w = weight of 1 cubic foot of water.
h = height of wall.
θ = angle of face of wall with horizon.

Problem II.

T = horizontal earth thrust per foot forward of wall, neglecting friction.

w_1 = weight of 1 cubic foot of earth.

h = height of wall.

H = horizontal earth thrust per foot forward of wall, taking friction into account.

ϕ = angle of repose of earth.

a = angle between wedge producing thrust and natural slope of earth.

Problem III.

π = 180°.

Notes on Problem III.

M = maximum horizontal earth thrust per foot forward of wall whose back is not vertical.

x = perpendicular let fall on natural slope from point where line of rupture cuts surface of ground.

c = length of line drawn at right angles to natural slope from lower point of back of wall to intersect surface of ground produced.

a = perpendicular let fall from top of back of wall on natural slope.

b = length of natural slope.

β = angle which natural slope makes with surface of ground.

Problem IV.

θ = angle which resultant makes with vertical.

w_2 = weight of 1 cubic foot of masonry.

w_1 = weight of 1 cubic foot of earth.

h = height of wall.

b = breadth of wall.

$\epsilon = \left(\frac{\pi}{4} - \frac{\phi}{2}\right)$

$q = \dfrac{\text{distance between centre of pressure and centre of base}}{\text{breadth of base}}$

Problem V.

b = breadth of base.

h = height of wall.

q = ratio of deviation of centre of pressure.

Problems VI. and VII.

a = breadth of wall at top.

b = breadth of wall at bottom.

h = height of wall.

w_1 = weight of 1 cubic foot of earth.

w_2 = weight of 1 cubic foot of masonry.

Problem VIII.

r = radius of battered face.

θ = $\frac{1}{2}$ angle subtended by curved face.

θ = circular measure of curved face.

Problem IX.

a = breadth of trapezoidal section at top.

b = breadth of base.

Problem X.

p = mean pressure per square foot of area.

m = maximum crushing force.

q = ratio of deviation of centre of pressure.

ADDENDA.

The Line of Resistance.

The above term is so often made use of in connection with retaining walls and other structures, that it may perhaps be as well thoroughly to explain its meaning, before leaving the theoretical portion of this work. The best mode of illustrating this point will be by a practical example.

Let A B C D (Fig. 22) be the cross section of a trapezoidal wall supporting a pressure of water against the face A B.

Take any plane E F, parallel to the base C B, and investigate the conditions of stability of the portion A F E D above it; thus,

Draw G L vertical passing through G, the centre of gravity of A F E D.

Draw M N horizontal, at a height M L = $\frac{1}{3}$ A F. Let K be the point where the resultant, produced by the pressure of water and weight of wall, cuts the plane E F.

Now the locus of the point K is the *line of resistance*, or, in other words, if any portion of the structure be separated from the rest by a horizontal plane and the point where the resultant pressure cuts this plane be determined, and called the *centre of pressure*, then the centre of pressure will always lie within a curve called the *line of resistance*.

The equation to the line of resistance may in this case be found as follows:

Let

w = weight of 1 cubic foot of water
w_2 = " " " masonry
h = height A F
a = breadth at top A D
b = " bottom E F
x = distance K F of centre of pressure from face of wall

Now

$$\frac{\text{K L}}{\text{L M}} = \frac{\text{pressure of water against A F}}{\text{weight of A F E D}}$$

$$\frac{3\,\text{K L}}{h} = \frac{h^2 w}{(a+b)\,h\,w_2}$$

$$\mathrm{K\,L} = h^2 \times \frac{1}{3\,(a+b)} \times \frac{w}{w_2}$$

$$x = \mathrm{K\,L} + \mathrm{L\,F}$$

$$x = h^2 \times \frac{1}{3\,(a+b)} \times \frac{w}{w_2} + \frac{a^2 + a\,b + b^2}{3\,(a+b)} \quad \text{(See Problem VI., Equation 32.)}$$

If the figure A B C D is a rectangle, $a = b$, and this equation becomes

$$x = h^2 \times \frac{1}{6\,a} \times \frac{w}{w^2} + \tfrac{1}{2}\,a$$

If the figure A B C D is a triangle, $a = 0$, and the equation becomes

$$x = h^2 \times \frac{1}{3\,b} \times \frac{w}{w_2} + \tfrac{1}{3}\,b$$

If in this last equation $\frac{w}{w_2} = \frac{b}{h}$, then $x = \frac{1}{3}\,h + \frac{1}{3}\,b$, or, expressed otherwise, if in a wall of triangular profile (supporting a pressure of water against its vertical face), the ratio of the breadth of base to height of wall be equal to that of weight of water to weight of masonry, then the line of resistance becomes a straight line and the section is one of theoretically uniform strength.

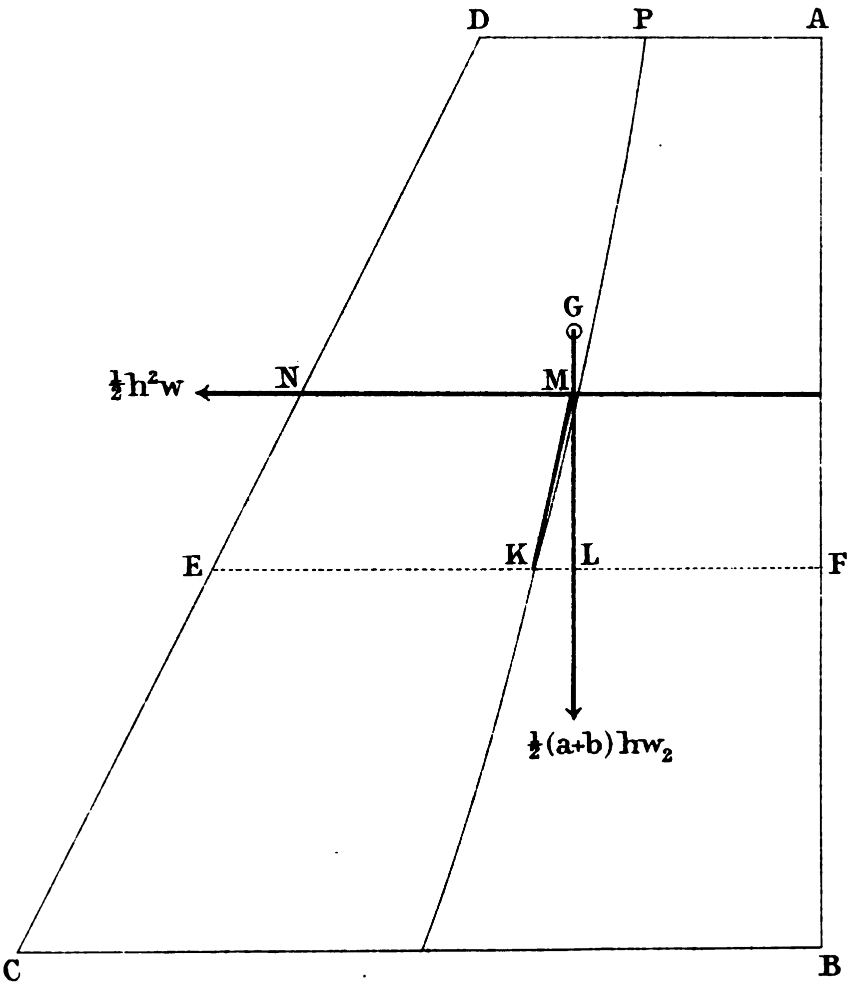
In the example shown on Fig. 22 the dimensions are as follows (drawn to a scale of 10 feet = 1 inch):

Height of wall	48 feet
Breadth of wall at top	18 "
Breadth of wall at bottom	42 "

and the line of resistance has been drawn by calculation on the assumption that the ratio of the weight of water to masonry is as 2 : 5, or that the masonry weighs 155 lbs. per cubic foot. It will be found that the line of resistance cuts the base C B almost exactly in the centre. This curve may be drawn geometrically by taking a series of horizontal lines and repeating the construction explained above for each. In doing this it may be useful to remember that the locus of L (the point lying directly under the centre of gravity of any section A D E F) is the straight line P L or P L produced, where P is the middle point of A D. Also the centres of gravity of all sections lie in the line joining the middle points of A D and B C.

In order that there may be no tension at any point of the structure, the

Fig. 22.



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line of resistance should always lie within the middle third of the breadth of the wall at that point.

The line of resistance determines the stability of position of the structure, in order to ensure which the distance between the point, where the line of resistance cuts any joint, and the middle point of the joint, must not exceed a certain fraction of the breadth of that joint (to be determined by practice). The frictional stability of the structure has however also to be taken into consideration. Now although the line of resistance determines the point at which the resultant pressure will cut any joint, it does not give the direction of this resultant.

Professor Moseley has therefore given the name of *line of pressure* to that curve to which the directions of all the resultant pressures are tangents.

The form of the line of pressure may be determined either analytically or geometrically, and when it is obtained in order to ensure the frictional stability of the structure, the normal to each joint must not make an angle greater than the angle of repose with a tangent to the line of pressure drawn through the centre of resistance of that joint.

On this subject consult

Rankine's 'Civil Engineering,' pp. 220, 416.
Moseley's 'Engineering,' p. 406.
Twisden's 'Mechanics,' p. 162.

Earth Pressure.

In all the problems relating to this subject it has been assumed that the line of rupture of a bank of earth supported by a wall is a straight line, and that the pressure is produced by earth acting in the form of a wedge. Whether this way of looking at the matter is as satisfactory as Rankine's more complicated method of reasoning may be open to question; but it is to be urged in favour of the wedge theory, that it illustrates the way in which the pressure *increases* as the amount of friction between the grains of earth *decreases*, in a way which is as near an approach to the real variation of pressure as the results of the more abstruse methods indicate.

The only way in which the real variations of pressure can be determined is by elaborate experiments on a larger scale than have yet been attempted.

The chief points to be kept in mind by the engineer are these, that

The greatest pressure that earth can under any circumstances produce

against the back of a wall is when the friction between its grains is destroyed and it assumes the form of mud.

The amount of this pressure is equal to that which would be produced by a fluid of the same specific gravity as the earth.

This pressure the author proposes to call *mud pressure*. The amount of mud pressure being a quantity which is absolutely certain and easily determinable, the engineer should always fix the pressure he intends his wall to be able to resist as a certain fraction of the mud pressure. The value of this fraction depends partly on the angle of repose of the earth supported, and partly on the amount of adhesion between the particles, and as has been stated above that though theory offers very plausible explanations more or less approximating to the truth, yet the only satisfactory way of determining the actual pressure is by experiment.

A safe value to assume for the ordinary pressure likely to be produced by common kinds of earth is $= \frac{1}{3}$ mud pressure, which according to the wedge theory corresponds to an angle of repose of 30°.

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